C H A P T E R 3 6 **Diffraction**

36-1 SINGLE-SLIT DIFFRACTION **6 – 1 SINGLE-SLIT DIFFRACTIO**

arning Objectives

Fresnang this module, you should be able to ...
 O1 Describe the diffraction of light waves by a narrow

and an edge, and also describe the resulting interfere
 O2 Des

Learning Objectives

After reading this module, you should be able to ...

- **36.01** Describe the diffraction of light waves by a narrow opening and an edge, and also describe the resulting interference pattern.
- 36.02 Describe an experiment that demonstrates the
- 36.03 With a sketch, describe the arrangement for a single-slit diffraction experiment.
- 36.04 With a sketch, explain how splitting a slit width into equal zones leads to the equations giving the angles to the minima in the diffraction pattern.
- 36.05 Apply the relationships between width a of a thin,

Key Ideas

● When waves encounter an edge, an obstacle, or an aperture 36.02 Describe an experiment that demonstrates the

single-slit diffraction experiment.

36.04 With a sketch, explain how splitting a slit width into

single-slit diffraction experiment.

36.04 With a sketch, explain how Fresnel bright spot.

Sien of With a sketch, describe the arrangement for a

single-slit diffraction experiment.

Sien Withing wavelength and the center of the diffraction

equal zones leads to the equations giving a slit interference. This type of interference is called diffraction.

 \bullet Waves passing through a long narrow slit of width a produce, on a viewing screen, a single-slit diffraction

- rectangular slit or object, the wavelength λ , the angle θ to any of the minima in the diffraction pattern, the distance to a viewing screen, and the distance between a minimum and the center of the pattern. From the minimal in the diffraction pattern,
and the diffraction pattern, the distance to
a viewing screen, and the diffraction pattern, the distance to
and the center of the pattern.
36.06 Sketch the diffraction pattern rectangular slit or object, the wavelength λ , the angle θ to any of the minima in the diffraction pattern, the distance to a viewing screen, and the distance between a minimum and the center of the pattern.
 06 Sk
- tifying what lies at the center and what the various bright and dark fringes are called (such as "first minimum").
- 36.07 Identify what happens to a diffraction pattern when aperture or object is varied.

pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles θ : what the various bright and
st minimum").
iffraction pattern when
e width of the diffracting
with the diffracting
immum (bright fringe) and other
imma that are located relative
1, 2, 3, ... (minima).
ately halfway between

 $a \sin \theta = m\lambda$, for $m = 1, 2, 3, ...$ (minima).

• The maxima are located approximately halfway between minima.

What Is Physics?

One focus of physics in the study of light is to understand and put to use the diffraction of light as it passes through a narrow slit or (as we shall discuss) past either a narrow obstacle or an edge.We touched on this phenomenon in Chapter 35 when we looked at how light flared—diffracted—through the slits in Young's experiment. Diffraction through a given slit of width a
produce, on a viewing screen, a single-slit diffraction
of the maxima are located approximate
with \mathbf{R} is more complicated to the maxima are located approximate **What Is Physics?**
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One focus of physics in the study of light is to understand and put to use the diffraction of light as it passes through a narrow slit or (as we shall discuss) past either a narrow o **What Is Physics?**
One focus of physics in the study of light is to understand and put to use the diffraction of light as it passes through a narrow slit or (as we shall discuss) past either a narrow obstacle or an edge. W **What Is Physics?**
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One focus of physics in the study of light is to understand and put to use the
diffraction of light as it passes through a narrow slit or (as we shall discuss) past
either a narrow obstacle or an edge. W tion applications worldwide is probably incalculable. focus of physics in the study of light is to understand and put to use the action of light as it passes through a narrow slit or (as we shall discuss) past er a narrow obstacle or an edge. We touched on this phenomenon in application opportunities. Even though the diffraction of light as it passes
through a slit or past an obstacle seems awfully academic, countless engineers
and scientists make their living using this physics, and the total

diffraction is due to the wave nature of light.

Diffraction and the Wave Theory of Light

In Chapter 35 we defined diffraction rather loosely as the flaring of light as it

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Ken Kay/Fundamental Photographs

Figure 36-1 This diffraction pattern appeared reached the screen. Diffraction caused the logical site of the edges, at both the edges, at both edges, and the screen. Produced the screen. Diffraction pattern a interference pattern consisting of a broad central maximum plus less intense and nar-Example 1997 Fundamental

Figure 36-1 This diffraction pattern appeared

Figure 36-1 This diffraction pattern appeared

on a viewing screen when light that had

passed through a narrow vertical slit

reached the screen. Di minima between them.

minimum intensity.

N
light produces an interference pattern called a **diffraction pattern.** For example, when
monochromatic light from a distant source (or a laser) passes through a narrow slit
and is then intercepted by a viewing screen, th monochromatic light from a distant source (or a laser) passes through a narrow slit and is then intercepted by a viewing screen, the light produces on the screen a diffrac-¹
 Solution partern in the partern called a **diffraction pattern.** For example, when

monochromatic light from a distant source (or a laser) passes through a narrow slit

and is then intercepted by a viewing screen, th bright) central maximum plus a number of narrower and less intense maxima (called Secondary or side maximal to both sides. The side may be monochromatic light from a distant source (or a laser) passes through a narrow slit and is then intercepted by a viewing screen, the light produces on the screen a d Fight produces an interference pattern called a **diffraction pattern**. For example, wh
monochromatic light from a distant source (or a laser) passes through a narrow s
and is then intercepted by a viewing screen, the light produces an interference pattern called a **diffraction pattern**. For example, when ochromatic light from a distant source (or a laser) passes through a narrow slit is then intercepted by a viewing screen, the light produce N
ight produces an interference pattern called a **diffraction pattern**. For example, when
monochromatic light from a distant source (or a laser) passes through a narrow slit
and is then intercepted by a viewing screen, th light produces an interference pattern called a **diffraction pattern**. For example, when
monochromatic light from a distant source (or a laser) passes through a narrow slit
and is then intercepted by a viewing screen, the

through to form a sharp rendition of the slit on the viewing screen instead of a conclude that geometrical optics is only an approximation. and is then intercepted by a viewing screen, the light produces on the screen a diffraction pattern like that in Fig. 36-1. This pattern consists of a broad and intense (very bright) central maximum plus a number of narrow

Edges. Diffraction is not limited to situations in which light passes through a tion pattern like that in Fig. 36-1. This pattern consists of a broad and intense (very
bright) central maximum plus a number of narrower and less intense maxima (called
secondary or **side** maxima to both sides. In betw bright) central maximum plus a number of narrower and less intense maxima (called **secondary** or **side** maxima) to both sides. In between the maxima are minima. Light flares into those dark regions, but the light waves can **secondary or side** maxima) to both sides. In between the maxima are minima. Light
flares into those dark regions, but the light waves cancel out one another.
Such a pattern would be totally unexpected in geometrical optic flares into those dark regions, but the light waves cancel out one another.

Such a pattern would be totally unexpected in geometrical optics: If light

traveled in straight lines as rays, then the slit would allow some of Such a pattern would be totally unexpected in geometrical optics: If light traveled in straight lines as rays, then the slit would allow some of those rays the pattern of bright and dark bands as we see in Fig. 36-1. As in traveled in straight lines as rays, then the slit would allow some of those rays
through to form a sharp rendition of the slit on the viewing screen instead of a
pattern of bright and dark bands as we see in Fig. 36-1. As geometrical optics prevailed. *Edges.* Diffraction is not limited to situations in which light passes through a
narrow opening (such as a slit or pinhole). It also occurs when light passes an
edge, such as the edges of the razor blade whose diffraction narrow opening (such as a slit or pinhole). It also occurs when light passes an edge, such as the edges of the razor blade whose diffraction pattern is shown in Fig. 36-2. Note the lines of maxima and minima that run appro

Floaters. You encounter a common example of diffraction when you look at a clear blue sky and see tiny specks and hairlike structures floating in your view.These are seeing when a floater is in your field of vision is the diffraction pattern produced Fig. 36-2. Note the lines of maxima and minima that run approximately parallel to
the edges, at both the inside edges of the blade and the outside edges. As the light
passes, say, the vertical edge at the left, it flares l the edges, at both the inside edges of the blade and the outside edges. As the light
passes, say, the vertical edge at the left, it flares left and right and undergoes inter-
ference, producing the pattern along the left e can distinguish individual maxima and minima in the patterns. nce, producing the pattern along the left edge. The rightmost portion of that
ern actually lies behind the blade, within what would be the blade's shadow if
metrical optics prevailed.
Floaters. You encounter a common exa pattern actually lies behind the blade, within what would be the blade's shadow if
geometrical optics prevailed.
Floaters. You encounter a common example of diffraction when you look at a
clear blue sky and see tiny specks **Folares.** You encounter a common example of diffraction when you look at a
 Floaters. You encounter a common example of diffraction when you look at a

clear blue sky and see tiny specks and hairlike structures floating **Floaters.** You encounter a common example of diffraction when you look at a clear blue sky and see tiny specks and hairlike structures floating in your view. These floaters, as they are called, are produced when light pas on a viewing screen when light that had *floaters*, as they are called, are produced when light passes the edges of tiny deposits passed through a narrow vertical slit in the vitreous humor, the transparent material filling most of the eyeball. What you light to flare out perpendicular to the long
on the retina by one of these deposits. If you sight through a pinhole in a piece of sides of the slit. That flaring produced an
interference perturn consisting of a broad cardboard so as to make the light entering your eye approximately a plane wave, you

because the sound waves diffract when they pass through the narrow opening of floaters, as they are called, are produced when light passes the edges of tiny deposits
in the vitreous humor, the transparent material filling most of the eyeball. What you
are seeing when a floater is in your field of vi in the vitreous humor, the transparent material filling most of the eyeball. What you
are seeing when a floater is in your field of vision is the diffraction pattern produced
cardboard so as to make the light netring your are seeing when a floater is in your field of vision is the diffraction pattern produced
on the retina by one of these deposits. If you sight through a pinhole in a piece of
cardboard so as to make the light entering your on the retina by one of these deposits. If you sight through a pinhole in a piece of cardboard so as to make the light entering your reye approximately a plane wave, you can distinguish individual maxima and minima in the of the sound reaches the fans in front of the cheerleader. wave and to occurs with outer types of waves as well. For example, you have produced a
haly seen diffraction in action at football games. When a cheerleader near the
playing field yells up at several thousand noisy fans, t signification in action at its olonal games. When a theerieater heat the
playing field yells up at several thousand noisy fans, the yell can hardly be heard
because the sound waves diffract when they pass through the narro playing tield yells up at several floudsaint lobisy lails, the yell can't interference the sound waves diffract when they pass through the nearrow opening of the cheerleader's mouth. This flaring leaves little of the waves

The Fresnel Bright Spot

because it ran counter to Newton's theory that light was a stream of particles.

Newton's view was the prevailing view in French scientific circles of the early 19th the lans in flott of the cheerleader. To onset the dinfaction, the cheerleader can
yell through a megaphone. The sound waves then emerge from the much wider
opening at the end of the megaphone. The flaring is thus reduced, in the wave theory of light, submitted a paper of the French Academy of the wave noise of the sound reaches the fans in front of the cheerleader.
 The Fresnel Bright Spot

Diffraction finds a ready explanation in the wav scribing his experiments with light and his wave-theory explanations of them. **Fresnel Bright Spot**
 Fresnel Bright Spot
 Fresnel Bright Spot
 Fresnel Bright Spot
 Contained by and used in the late 1600s by Huygens and used 123 years later by

orginally advanced in the late 1600s by Huygens

The Fresnel Bright Spot

Diffraction finds a ready explanation in the wave theory of light. However, this theory, originally advanced in the late 1600s by Huygens and used 123 years later by

Young to explain double-slit **Diffraction** finds a ready explanation in the wave theory of light. However, this theory, originally advanced in the late 1600s by Huygens and used 123 years later by Young to explain double-slit interference, was very sl Diffraction finds a ready explanation in the wave theory of light. However, this theory, originally advanced in the late 1600s by Huygens and used 123 years later by Young to explain double-slit interference, was very slow ory, originally advanced in the late 1600s by Huygens and used 123 years later by

Young to explain double-slit interference, was very slow in being adopted, largely

because it ran counter to Newton's theory that light wa Young to explain double-slit interference, was very slow in being adopted, largely because it ran counter to Newton's theory that light was a stream of particles. The weak the prevailing view in French scientific circles o because it ran counter to Newton's theory that light was a stream of particles.
Newton's view was the prevailing view in French scientific circles of the early 19th
century, when Augustin Fresnel was a young military engin Ken Kay/Fundamental Photographs subject of diffraction. Fresnel won. The Newtonians, however, were not swayed. Figure 36-2 The diffraction pattern produced One of them, S. D. Poisson, pointed out the "strange result" that if Fresnel's theories by a razor blade in monochromatic light. were correct, then light waves should flare into the shadow region of a sphere Note the lines of alternating maximum and as they pass the edge of the sphere, producing a bright spot at the center of

36-1 SINGLE-SLIT DIFFR
covered that the predicted *Fresnel bright spot*, as we call it today, was indeed there
(Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unex-
pected and counterintuiti 36-1

covered that the predicted *Fresnel bright spot*, as we call it today, was indeed there

(Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unex-

pected and counterintuitive predictions pected and counterintuitive predictions verified by experiment.

Diffraction by a Single Slit: Locating the Minima

Let us now examine the diffraction pattern of plane waves of light of wavelength 36-1 SINGLE-SLIT DI

covered that the predicted *Fresnel bright spot*, as we call it today, was indeed there

(Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unex-

pected and counterintuit covered that the predicted *Fresnel bright spot*, as we call it today, was indeed there (Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unexpected and counterintuitive predictions verified covered that the predicted *Fresnel bright spot*, as we call it today, was indeed there (Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unexpected and counterintuitive predictions verified the screen and the predicted *Fresnel bright spot*, as we call it today, was indeed there (Fig. 36-3). Nothing builds confidence in a theory so much as having one of its unexpected and counterntuitive predictions verified different points within the slit undergo interference and produce a diffraction pattern of bright and dark fringes (interference maxima and minima) on the **Diffraction by a Single Slit: Locating the Minima**
Let us now examine the diffraction pattern of plane waves of light of wavelength λ that are diffracted by a single long, narrow slit of width *a* in an otherwise opaq **Diffraction by a Single Slit: Locating the Minima**

Let us now examine the diffraction pattern of plane waves of light of wavelength
 λ that are diffracted by a single long, narrow slit of width a in an otherwise

len **Diffraction by a Single Slit: Locating the Minima**

Let us now examine the diffraction pattern of plane waves of light of wavelength
 λ that are diffracted by a single long, narrow slit of width *a* in an otherwise

o equations for only the dark fringes. as now examine the diffraction pattern of plane waves of light of wavelength
at are diffracted by a single long, narrow slit of width a in an otherwise
que screen B, as shown in cross section in Fig. 36-4. (In that figure λ that are diffracted by a single long, narrow slit of width *a* in an otherwise opaque screen *B*, as shown in cross section in Fig. 36-4. (In that figure, the slit's length extends into and out of the page, and the i length extends into and out of the page, and the incoming wavefronts are parallel
to screen *B*.) When the diffracted light reaches viewing screen *C*, waves from
different points within the slit undergo interference and p From points within the slit undergo interference and produce a diffraction

Parm of bright and dark fringes (interference maxima and minima) on the

metaston. To locate the fringes in a two-slit interference pattern. Howe

about the same distance to reach the center of the pattern and thus are in phase halfway between adjacent dark fringes.

strategy that involves pairing up all the rays coming through the slit and then finding what conditions cause the wavelets of the rays in each pair to cancel each one we used to locate the fringes in a two-slit interference pattern. However,
diffraction ings and the Fre
equations for only the dark fringes, and here we shall be able to find
equations from all points are equations of diffraction is more mathematically challenging, and here we shall be able to find
equations for only the dark fringes.
Hence, we can justify the central bright fringe seen in
the second the slit into the slit into the sli tend to P_1 a light ray r_1 from the top point of the top zone and a light ray r_2 from Before we do that, however, we can justify the central bright fringe seen in

Fig. 36-1 by noting that the Huygens wavelets from all points in the slit travel

about the same distance to reach the center of the pattern an cancel each other when they arrive at P_1 . Then any similar pairing of rays from velets from all points in the slit travel

ter of the pattern and thus are in phase

an say only that they are approximately

ve shall use a clever (and simplifying)

ve shall use a clever (and simplifying)

rays coming t the two zones will give cancellation.A central axis is drawn from the center of the there. As for the other bright fringes, we can say only tha
halfway between adjacent dark fringes.
Pairings. To find the dark fringes, we shall use a
strategy that involves pairing up all the rays coming th
finding what slit to screen C, and P_1 is located at an angle θ to that axis. strategy that involves pairing up all the rays coming through the slit and then
finding what conditions cause the wavelets of the rays in each pair to cancel each
other. We apply this strategy in Fig. 36-4 to locate the f

Path Length Difference. The wavelets of the pair of rays r_1 and r_2 are in $\left|\right|$ phase within the slit because they originate from the same wavefront passing fringe they must be out of phase by $\lambda/2$ when they reach P_1 ; this phase difference other. We apply this strategy in Fig. 36-4 to locate the first dark fringe, at point P_1 .
First, we mentally divide the slit into two *zones* of equal widths a/2. Then we ex-
tend to P_1 a light ray r_1 from the top to reach P_1 being longer than the path traveled by the wavelet of r_1 . To display tend to P_1 a light ray r_1 from the top point of the top zone and a light ray *i* the top point of the bottom zone. We want the wavelets along these two cancel each other when they arrive at P_1 . Then any similar p this path length difference, we find a point b on ray r_2 such that the path length $a/2$ from b to P_1 matches the path length of ray r_1 . Then the path length difference between the two rays is the distance from the center of the slit to b. Wo zones will give cancellation. A central axis is drawn from the center of the

o screen C, and P_1 is located at an angle θ to that axis.

The article correct correct correct correct correct correct correct correct slit to screen C, and P_1 is located at an angle θ to that axis.
 Path Length Difference. The wavelets of the pair of rays r_1 and r_2 are in

through the slit, leaves they originate from the same wavefront pas phase within the slit because they originate from the same wavefront passing
through the slit, along the width of the slit. However, to produce the first dark
fringe they must be out of phase by $\lambda/2$ when they reach $P_$

mathematics considerably if we arrange for the screen separation D to be much
Figure 36-4 Waves from the top points of two

Courtesy Jearl Walker

Figure 36-3 A photograph of the diffraction diffraction rings and the Fresnel bright Courtesy Jearl Walker
 Figure 36-3 A photograph of the diffraction

pattern of a disk. Note the concentric

diffraction rings and the Fresnel bright

spot at the center of the pattern. This

experiment is essentially ide Courtesy Jearl Walker
 Figure 36-3 A photograph of the diffraction

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Figure 36-3 A photograph of the diffraction
pattern of a disk. Note the concentric
diffraction rings and the Fresnel bright
spot at the center of the pattern. This
experiment is essentially identical they used and the disk used here have a cross section with a circular edge.

and r_2 zones of width a/2 undergo fully destructive interference at point P_1 on viewing screen C.

of four zones of width a/4 undergo fully $,r_2,r_3$, and r_4 as being parallel, at angle θ to the central axis.

A
as being parallel, at angle θ to the central axis. We can also approximate the trian-
gle formed by point b , the top point of the slit, and the center point of the slit as
being a right triangle, and one of the ang as being parallel, at angle θ to the central axis. We can also approximate the trian-
gle formed by point *b*, the top point of the slit, and the center point of the slit as
being a right triangle, and one of the angle as being parallel, at angle θ to the central axis. We can also approximate the triangle formed by point b , the top point of the slit, and the center point of the slit as being a right triangle, and one of the angles path length difference between rays r_1 and r_2 (which is still the distance from the center of the slit to point b) is then equal to $(a/2)$ sin θ . as being parallel, at angle θ to the central axis. We can also approximate the trian-
gle formed by point *b*, the top point of the slit, and the center point of the slit as
being a right triangle, and one of the angle e central axis. We can also approximate the trian-
oint of the slit, and the center point of the slit as
f the angles inside that triangle as being θ . The
ays r_1 and r_2 (which is still the distance from the
n equ as being parallel, at angle θ to the central axis. We can also approximate the trian-
gle formed by point *b*, the top point of the slit, and the center point of the slit as
being a right triangle, and one of the angle as being parallel, at angle θ to the central axis. We can also approximate the triangle formed by point b , the top point of the slit, and the center point of the slit a being a right triangle, and one of the angles i as being parallel, at angle θ to the central axis. We can also approximate the triangle formed by point b , the top point of the slit, and the center point of the slit as

First Minimum. We can repeat this analysis for any other pair of rays origizones) and extending to point P_1 . Each such pair of rays has the same path length First Minimum. We can repeat this analysis for any other pair of rays origi-

First Minimum. We can repeat this analysis for any other pair of rays origi-

rating at corresponding points in the two zones (say, at the midp (our condition for the first dark fringe), we have

 $a \cdot a \lambda$

which gives us

 $a \sin \theta = \lambda$ (first minimum). (36-1)

 $\frac{a}{2}\sin\theta=\frac{\lambda}{2},$ 2' ,

fringe above and (by symmetry) below the central axis.

Narrowing the Slit. Note that if we begin with $a > \lambda$ and then narrow the slit zones) and extending to point P_1 . Each such pair of rays has the same pair length
difference (a/2) sin θ . Setting this common path length difference equal to $\lambda/2$
(our condition for the first dark fringe), we have difference (a/2) sin θ . Setting this common pain length dimerence equal to λ /2

(our condition for the first dark fringe), we have
 $\frac{a}{2} \sin \theta = \frac{\lambda}{2}$,

which gives us
 $a \sin \theta = \lambda$ (first minimum). (36-1)

Given s and the width of the pattern) is greater for a narrower slit.When we have reduced the slit width to the wavelength (that is, $a = \lambda$), the angle of the first dark fringes λ

(36-1)

1 tells us the angle θ of the first dark

ntral axis.

m with *a* > λ and then narrow the slit

increase the angle at which the first

diffraction (the extent of the flaring
 narrower slit. When we have red which gives us
 $\frac{1}{2} \sin \theta = \frac{1}{2}$,
 $\sin \theta = \lambda$ (first minimum). (36-1)

Given slit width *a* and wavelength λ , Eq. 36-1 tells us the angle θ of the first dark

fringe above and (by symmetry) below the central axi that bright fringe must then cover the entire viewing screen. Given slit width *a* and wavelength λ , Eq. 36-1 tells us the angle θ of the first dark
fringe above and (by symmetry) below the central axis.
Narrowing the Slit. Note that if we begin with $a > \lambda$ and then narrow t Given slit width a and wavelength λ , Eq. 36-1 tells us the angle θ of the first dark

fringe above and (by symmetry) below the central axis.
 Narrowing the Slit. Note that if we begin with $a > \lambda$ and then narrow t ge above and (by symmetry) below the cen
Narrowing the Slit. Note that if we begin
e holding the wavelength constant, we in
fringes appear; that is, the extent of the d
the width of the pattern) is *greater* for a *nc*
 and then narrow the slit
angle at which the first
(the extent of the flaring
. When we have reduced
e of the first dark fringes
the central bright fringe,
en.
s above and below the
at we now divide the slit
. We then exte

Second Minimum. We find the second dark fringes above and below the , r_2 , r_3 , and r_4 from the top points of the zones to point P_2 , the location of the **Narrowing the Slit.** Note that if we begin with $a > \lambda$ and then narrow the slit while holding the wavelength constant, we increase the angle at which the first dark fringes appear; that is, the extent of the diffraction ference between r_1 and r_2 , that between r_2 and r_3 , and that between r_3 and r_4 must mgth constant, we increase the angle at v
is, the extent of the diffraction (the extent
ern) is *greater* for a *narrower* slit. When we
length (that is, $a = \lambda$), the angle of the firs
fringes mark the two edges of the ce ease the angle at which the first

fraction (the extent of the flaring
 ower slit. When we have reduced

the angle of the first dark fringes

dges of the central bright fringe,
 ing screen.
 k fringes above and belo all be equal to $\lambda/2$. the width of the pattern) is *greater* for a *narrower* slit. When we have reduced
slit width to the wavelength (that is, $a = \lambda$), the angle of the first dark fringes
e. Since the first dark fringes mark the two edges of the slit width to the wavelength (that is, $a = \lambda$), the angle of the first dark fringes
is 90°. Since the first dark fringes mark the two edges of the central bright fringe,
that bright fringe must then cover the entire v is 90°. Since the first dark fringes mark the two edges of the central bright fringe,
that bright fringe must then cover the entire viewing screen.
Second Minimum. We find the second dark fringes above and below the
Se that bright fringe must then cover the entire viewing screen.
 Second Minimum. We find the second dark fringes above and below the central axis as we found the first dark fringes, except that we now divide the slit into

Second Minimum. We find the second dark fringes above and below the central axis as we found the first dark fringes, except that we now divide the slit into four zones of equal widths $a/4$, as shown in Fig. 36-6a. We t We see from the top triangle that the path length difference between r_1 and r_2 is is central axis as we found the first dark fringes, except that we now divide the slit
into *four* zones of equal widths $a/4$, as shown in Fig. 36-6*a*. We then extend rays r_1 ,
 r_2 , r_3 , and r_4 from the top points r_3 and r_4 is also (a/4) sin θ . In zones of equal widths $a/4$, as shown in Fig. 36-6a. We then extend rays r_1 , r_4 from the top points of the zones to point P_2 , the location of the sec-
fringe above the central axis. To produce that fringe, the pa r_2 , r_3 , and r_4 from the top points of the zones to point P_2 , the location of the second dark fringe above the central axis. To produce that fringe, the path length diference between r_1 and r_2 , that betwe ond dark fringe above the central axis. To produce that fringe, the path length dif-
ference between r_1 and r_2 , that between r_2 and r_3 , and that between r_3 and r_4 must
all be equal to $\lambda/2$.
For $D \ge a$, we see no the top that the path length difference between t_1 and t_2 is $(a/4)$ sin θ . Similarly, from the bottom triangle, the path length difference for any two rays that originate at corresponding points in two Figure 36-6 (*a*) Waves from the top points r_3 and r_4 is also (*a*/4) sin θ . In fact, the path length difference for any two rays that of four zones of width *a*/4 undergo fully originate at corresponding points destructive interference at point P_2 . (b) For each such case the path length difference is equal to $\lambda/2$, we have ond dark fringe above the ce
ference between r_1 and r_2 , th
all be equal to $\lambda/2$.
For $D \ge a$, we can approve the central axis. To display t
cular line through each adja
ries of right triangles, each
We see from the

$$
\frac{a}{4}\sin\,\theta=\frac{\lambda}{2},
$$

which gives us

$$
a\sin\theta = 2\lambda \qquad \text{(second minimum)}.\tag{36-2}
$$

All Minima. We could now continue to locate dark fringes in the diffraction choose an even number of zones so that the zones (and their waves) could be *p*₃ and *r*₄ is also (*u*/4) sin *b*. In fact, the pain length difference for any two rays that originate at corresponding points in two adjacent zones is (*a*/4) sin *θ*. Since in each such case the path length diff low the central axis can be located with the general equation $\frac{a}{4} \sin \theta = \frac{\lambda}{2}$,
 $a \sin \theta = 2\lambda$ (second minimum).

We could now continue to locate dark fringes in tg up the slit into more zones of equal width. We umber of zones so that the zones (and their wa been doing. We would $\frac{1}{2}$ sin $\theta = \frac{\lambda}{2}$,

(second minimum). (36-2)

Intinue to locate dark fringes in the diffraction

more zones of equal width. We would always

so that the zones (and their waves) could be

would find that the dark *a* sin $\theta = 2\lambda$ (second minimum). (36-2)
 All Minima. We could now continue to locate dark fringes in the diffraction

ern by splitting up the slit into more zones of equal width. We would always

sose an even number a sin $\theta = 2\lambda$ (second minimum). (36-2)
 All Minima. We could now continue to locate dark fringes in the diffraction

pattern by splitting up the slit into more zones of equal width. We would always

choose an even num **All Minima.** We could now continue to locate dark fringes in the diffraction pattern by splitting up the slit into more zones of equal width. We would always choose an even number of zones so that the zones (and their wa

$$
a \sin \theta = m\lambda
$$
, for $m = 1, 2, 3, ...$ (minima—dark fringes). (36-3)

In a single-slit diffraction experiment, dark fringes are produced where the path length differences (*a* sin θ) between the top and bottom rays are equal to λ , 2λ , 3λ , ... In a single-slit diffraction experiment, dark fringes are produced where the
path length differences (*a* sin θ) between the top and bottom rays are equal
to λ , 2 λ , 3 λ ,
may seem to be wrong because the wa In a single-slit diffraction experiment, dark fringes are procepath length differences $(a \sin \theta)$ between the top and botto to λ , 2λ , 3λ ,
may seem to be wrong because the waves of those two ply in phase with e

This may seem to be wrong because the waves of those two particular rays will be exactly in phase with each other when their path length difference is an integer **1988**
 1988 In a single-slit diffraction experiment, dark fringes are produced where the

path length differences $(a \sin \theta)$ between the top and bottom rays are equal

to λ , 2λ , 3λ ,

This may seem to be wro that are exactly out of phase with each other; thus, each wave will be canceled by Sometimes of the same of wave seriance of α and β) between the top and bottom rays are equal to λ , 2 λ , 3 λ ,
This may seem to be wrong because the waves of those two particular rays will be exactly in ph In a single-slit diffraction experiment, dark fringes are produced where the path length differences (*a* sin θ) between the top and bottom rays are equal to λ , 2λ , 3λ ,
This may seem to be wrong because t to be exactly in phase with other light waves.) In a single-slit diffraction experiment, dark fringes are produced where the
path length differences (*a* sin θ) between the top and bottom rays are equal
to λ , 2λ , 3λ ,....
This may seem to be wrong because th This may seem to be wrong because the waves of those two particular rays will be exactly in phase with each other when their path length difference is an integer number of wavelengths. However, they each will still be par

ing screen and then move the screen in so that it coincides with the focal plane of the parallel (rather than approximately) back at the slit.They are like the initially parallel rays of Fig.34-14a that are directed to the focal point by a converging lens. be exactly in phase with other light waves.)

Using a Lens. Equations 36-1, 36-2, and 36-3 are derived for the case of $D \ge a$.

owever, they also apply if we place a converging lens between the slit and the view-

g scree (a) switch to yellow light or (b) decrease the slit width?

The parameter of the street in so that it coincides with the focal p

is g screen and then move the screen in so that it coincides with the focal p

is an S. The

Checkpoint 1

We produce a diffraction pattern on a viewing screen by means of a long narrow (the maxima and minima shift away from the center) or contract toward it if we ffraction pattern on a viewing screen by means of
by blue light. Does the pattern expand away from the
d minima shift away from the center) or contract to
low light or (b) decrease the slit width?
blem 36.01 Single-slit

Sample Problem 36.01 Single-slit diffraction pattern with white light

A slit of width a is illuminated by white light.

KEY IDEA

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15^{\circ}$?

KEY IDEA

Diffraction occurs separately for each wavelength in the **Sample Problem 36.01 Single-slit diffraction pattern with white light**
A slit of width *a* is illuminated by white light.
(a) For what value of *a* will the first minimum for red light of
wavelength $\lambda = 650$ nm appear a Sample Problem 36.01 Single-slit diffraction pattern with white light

A slit of width *a* is illuminated by white light.

(a) For what value of *a* will the first minimum for red light of

wavelength $\lambda = 650$ nm appear $(a \sin \theta = m\lambda).$ (a) For what value of a will the first minimum for red light of
wavelength $\lambda = 650$ nm appear at $\theta = 15^{\circ}$?
 KEY IDEA

Diffraction occurs separately for each wavelength in the first side maximum

Diffraction occurs

Calculation: When we set $m = 1$ (for the first minimum)

$$
a = \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15^{\circ}}
$$

= 2511 nm \approx 2.5 \mu \text{m}. (Answer)

Diffraction occurs separately for each wavelength in the the first side maximum at range of wavelengths passing through the slit, with the loca-

(a sin $\theta = m\lambda$).

(a sin $\theta = m\lambda$).

Calculation: When we set $m = 1$ (for range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36-3

(a sin $\theta = m\lambda$).

Calculation: When we set $m = 1$ (for the first minimum)

Solving for λ' and subs tions of the minima for each wavelength given by Eq. 36-3

(a sin θ = m λ).
 Calculation: When we set $m = 1$ (for the first minimum)

Solving for λ' and substitute

and substitute the given values of θ and $\$ (*a* sin $\theta = m\lambda$).
 Calculation: When we set $m = 1$ (for the first minimum)

and substitute the given values of θ and λ , Eq. 36-3 yields
 $a = \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15^\circ}$
 $= 2511 \text{ nm} \approx 2.5 \mu \text{m}.$ (Answe and substitute the given values of θ and λ , Eq. 36-3 yields
 $a = \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15^\circ}$
 $= 2511 \text{ nm} \approx 2.5 \mu \text{m}.$ (Answer) Light of

For the incident light to flare out that much (±15° to the first wa

minimum for the red light?

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

Calculations: Those first and second minima can be located **EXEMPT WE SET THE SET ASSET SET ASSET ASSET SET ASSET THE first side maximum for any wavelength is about half
between the first and second minima for that wavelength.
Calculations:** Those first and second minima can be with Eq. 36-3 by setting $m = 1$ and $m = 2$, respectively. Thus, gth is about halfway

that wavelength.

inima can be located

2, respectively. Thus,

d *approximately* by the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 36-3 becomes 1.5.Then Eq. 36-3 becomes ^a sin ^u e maximum for any wavelength is about half irst and second minima for that wavelengt
 s: Those first and second minima can be loce 3 by setting $m = 1$ and $m = 2$, respectively.⁷

le maximum can be located *approximate*

$$
a\sin\theta=1.5\lambda'.
$$

Solving for λ' and substituting known data yield

$$
\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^{\circ})}{1.5}
$$

= 430 nm. (Answer)

with Eq. 36-3 by setting $m = 1$ and $m = 2$, respectively. Thus,
the first side maximum can be located *approximately* by
setting $m = 1.5$. Then Eq. 36-3 becomes
 $a \sin \theta = 1.5\lambda'$.
Solving for λ' and substituting known da where the first side maximum can be located *approximately* by
setting $m = 1.5$. Then Eq. 36-3 becomes
 $a \sin \theta = 1.5\lambda'$.
Solving for λ' and substituting known data yield
 $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$
= 4 setting $m = 1.5$. Then Eq. 36-3 becomes
 $a \sin \theta = 1.5\lambda'$.

Solving for λ' and substituting known data yield
 $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$
 $= 430 \text{ nm}.$ (Answer)

Light of this wavelength is violet (far Solving for λ' and substituting known data yield
 $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$

= 430 nm. (Answer)

Light of this wavelength is violet (far blue, near the short-

wavelength limit of the human range of a sin $\theta = 1.5\lambda'$.

Solving for λ' and substituting known data yield
 $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$

= 430 nm. (Answer)

Light of this wavelength is violet (far blue, near the short-

wavelength limit Solving for λ' and substituting known data yield
 $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$

= 430 nm. (Answer)

Light of this wavelength is violet (far blue, near the short-

wavelength limit of the human range of $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^{\circ})}{1.5}$
= 430 nm. (Answer)
Light of this wavelength is violet (far blue, near the short-
wavelength limit of the human range of visible light). From
the two equations we used, can yo $\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15^\circ)}{1.5}$
= 430 nm. (Answer)
Light of this wavelength is violet (far blue, near the short-
wavelength limit of the human range of visible light). From
the two equations we used, can you conversely. Light of this wavelength is violet (far blue, near the short-(b) What is the wavelength λ' of the light whose first side which this overlap occurs does depend on slit width. If the

 $\overleftrightarrow{PLUS}$ Additional examples, video, and practice available at WileyPLUS

36-2 INTENSITY IN SINGLE-SLIT DIFFRACTION

Learning Objectives

After reading this module, you should be able to . . .

- 36.08 Divide a thin slit into multiple zones of equal width **86** CHAPTER 36 DIFFRACTION
 6 - **2** INTENSITY IN SINGLE-SLIT DIFFRACTION

arring Objectives

For reading this module, you should be able to ...
 08 Divide a thin slit into multiple zones of equal width

and write an wavelets from adjacent zones in terms of the angle θ to a point on the viewing screen.
- 36.09 For single-slit diffraction, draw phasor diagrams for **6 – 2** INTENSITY IN SINGLE-SLIT DIFFRACTI

er reading this module, you should be able to ...
 08 Divide a thin slit into multiple zones of equal width and write an expression for the phase difference of the **36.10**

wa to one side, indicating the phase difference between adjacent phasors, explaining how the net electric field is calculated, and

Key Idea

• The intensity of the diffraction pattern at any given angle θ is where I_m is the $I(\theta) = I_m \left(\frac{\sin \alpha}{\theta} \right)^2$,

$$
I(\theta) = I_m \bigg(\frac{\sin \alpha}{\alpha} \bigg)^2,
$$

identifying the corresponding part of the diffraction pattern.

- 36.10 Describe a diffraction pattern in terms of the net electric field at points in the pattern.
- **36.11** Evaluate α , the convenient connection between angle θ to a point in a diffraction pattern and the intensity I at that point.
- 36.12 For a given point in a diffraction pattern, at a given angle, calculate the intensity I in terms of the intensity I_m at the center of the pattern.

where I_m is the intensity at the center of the pattern and

$$
\alpha = \frac{\pi a}{\lambda} \sin \theta.
$$

Intensity in Single-Slit Diffraction, Qualitatively

In Module 36-1 we saw how to find the positions of the minima and the maxima in a single-slit diffraction, **Qualitatively**

in a single-**Slit Diffraction, Qualitatively**

In Module 36-1 we saw how to find the positions of the minima and the maxima

in a single-slit diffraction pattern. Now we turn strategy of the intensity at the center of the pattern and
 $\alpha = \frac{\pi a}{\lambda} \sin \theta$.
 In tensity in Single-Slit Diffraction, Qualitatively

In Module 36-1 we saw how to find the positions of the minima and the maxima

in a s tion of a point on a viewing screen. **Partical Solution Single-Slit Diffraction, Qualitatively**
 Photon Single-Slit Diffraction, Qualitatively
 Photon Single-Slit of Fig. 36-4 into a more general problem: find

xpression for the intensity *I* of the patt

Intensity in Single-Slit Diffraction, Qualitatively
In Module 36-1 we saw how to find the positions of the minima and the maxima
in a single-slit diffraction pattern. Now we turn to a more general problem: find
an expre wish to superimpose the wavelets arriving at an arbitrary point P on the viewing **Intensity in Single-Slit Diffraction, Qualitatively**
In Module 36-1 we saw how to find the positions of the minima and the maxima
in a single-slit diffraction pattern. Now we turn to a more general problem: find
an expre the electric component of the resultant wave at P . The intensity of the light at P is then proportional to the square of that amplitude. 1 we saw how to find the positions of the minima and the maxima diffraction pattern. Now we turn to a more general problem: find for the intensity *I* of the pattern as a function of θ , the angular position a viewing s in a single-slit diffraction pattern. Now we turn to a more general problem: find
an expression for the intensity I of the pattern as a function of θ , the angular posi-
tion of a point on a viewing screen.
To do this,

To find E_{θ} , we need the phase relationships among the arriving wavelets. The point here is that in general they have different phases because they travel different is given by wish to superimpose the wavelets arriving at an arbitrary point *P* on the viewing
screen, at angle θ to the central axis, so that we can determine the amplitude E_{θ} of
the electric component of the resultant wave screen, at angle θ to the central axis, so that we can determine the amplitude E_{θ} of
the electric component of the resultant wave at *P*. The intensity of the light at *P* is
then proportional to the square of tha

$$
\begin{pmatrix} \text{phase} \\ \text{difference} \end{pmatrix} = \left(\frac{2\pi}{\lambda}\right) \begin{pmatrix} \text{path length} \\ \text{difference} \end{pmatrix}.
$$

adjacent zones as

$$
\Delta \phi = \left(\frac{2\pi}{\lambda}\right) (\Delta x \sin \theta). \tag{36-4}
$$

We assume that the wavelets arriving at P all have the same amplitude ΔE . is given by
 $\left(\begin{array}{c}\text{phase} \\ \text{difference}\end{array}\right) = \left(\frac{2\pi}{\lambda}\right) \left(\begin{array}{c}\text{path length} \\ \text{difference}\end{array}\right).$

For point *P* at angle *θ*, the path length difference between wavelets from adjacent

zones is $\Delta x \sin \theta$. Thus, we can write the phase di $\left(\begin{array}{c}\text{phase}\end{array}\right) = \left(\frac{2\pi}{\lambda}\right) \left(\begin{array}{c}\text{path length}\end{array}\right).$
For point *P* at angle *θ*, the path length difference between wavelets from adjacent zones is $\Delta x \sin \theta$. Thus, we can write the phase difference $\Delta \phi$ between wav the wavelet from each zone in the slit. on the characteristic term adjacent

ifference $\Delta \phi$ between wavelets from

sin θ). (36-4)

2 all have the same amplitude ΔE .

at P, we add the amplitudes ΔE via

f N phasors, one corresponding to

0 on the centra

Central Maximum. For point P_0 at $\theta = 0$ on the centra For point *P* at angle θ , the path length difference between wavelets from adjacent
zones is $\Delta x \sin \theta$. Thus, we can write the phase difference $\Delta \phi$ between wavelets from
adjacent zones as
 $\Delta \phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin \theta$ zones is Δx sin θ . Thus, we can write the phase difference $\Delta \phi$ between wavelets from
adjacent zones as
 $\Delta \phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin \theta)$. (36-4)
We assume that the wavelets arriving at P all have the same amplitude adjacent zones as
 $\Delta \phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin \theta)$. (36-4)

We assume that the wavelets arriving at *P* all have the same amplitude ΔE .

To find the amplitude E_{θ} of the resultant wave at *P*, we add the amplitudes $\Delta \phi = \left(\frac{2\pi}{\lambda}\right) (\Delta x \sin \theta).$ (36-4)
We assume that the wavelets arriving at *P* all have the same amplitude ΔE .
To find the amplitude E_{θ} of the resultant wave at *P*, we add the amplitudes ΔE via
phasors. To d $\Delta \phi = \left(\frac{\Delta \phi}{\lambda}\right) (\Delta x \sin \theta)$. (36-4)
We assume that the wavelets arriving at *P* all have the same amplitude ΔE .
To find the amplitude E_{θ} of the resultant wave at *P*, we add the amplitudes ΔE via
phasors. To

wave at P_0 is the vector sum of these phasors. This arrangement of the phasors turns out to be the one that gives the greatest value for the amplitude E_{θ} . We call this value E_m ; that is, E_m is the value of E_θ for $\theta = 0$. 0.

We next consider a point P that is at a small angle θ to the central axis. Equation 36-4 now tells us that the phase difference $\Delta \phi$ between wavelets from screen lying at a small angle θ to the central axis, (c)
the first minimum, and (d) the first side maximum.
Wave at P_0 is the vector sum of these phasors. This arrangement of the phasors
turns out to be the one that the first minimum, and (*d*) the first side maximum.

wave at P_0 is the vector sum of these phasors. This arrangement of the phasors

turns out to be the one that gives the greatest value for the amplitude E_{θ} . We angle $\Delta\phi$ between adjacent phasors. The amplitude E_θ at this new point is still the wave at P_0 is the vector sum of these phasors. This arrangement of the phasors
turns out to be the one that gives the greatest value for the amplitude E_{θ} . We call
this value E_m ; that is, E_m is the value of $E_{$ that the intensity of the light is less at this new point P than at P_0 .
First Minimun. If we continue to increase θ , the angle $\Delta \phi$ between adjacent pha-. e at P_0 is the vector sum of these phasors. This arrangement of the phasors s out to be the one that gives the greatest value for the amplitude E_{θ} . We call value E_m ; that is, E_m is the value of E_{θ} for θ wave at P_0 is the vector sum of these phasors. This arrangement of the phasors
turns out to be the one that gives the greatest value for the amplitude E_θ . We call
this value E_m ; that is, E_m is the value of E_θ turns out to be the one that gives the greatest value for the amplitude E_{θ} . We call this value E_m ; that is, E_m is the value of E_{θ} for $\theta = 0$. We next consider a point *P* that is at a small angle θ to th

tude E_{θ} is now zero, which means that the intensity of the light is also zero. We have is now zero, which means that the intensity of the light is also zero. We have
the first minimum, or dark fringe, in the diffraction pattern. The first and last
now have a phase difference of 2π rad, which means that t The *E₀* is now zero, which means that the intensity of the light is also zero. We have reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of 2 tude E_{θ} is now zero, which means that the intensity of the light is also zero. We have reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of ference between the top and bottom rays through the slit equals one wavelength. Recall that this is the condition we determined for the first diffraction minimum. A

attive E_{θ} is now zero, which means that the intensity of the light is also zero. We have

reached the first minimum, or dark fringe, in the diffraction pattern. The first and last

phasors now have a phase differe

First Side Maximum. As we continue to increase θ , the angle $\Delta \phi$ between A

dude E_{θ} is now zero, which means that the intensity of the light is also zero. We have

reached the first minimum, or dark fringe, in the diffraction pattern. The first and last

phasors now have a phase differenc **idde** E_{θ} **is now zero, which means that the intensity of the light is also zero. We have reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of** ment corresponds to the first side maximum in the diffraction pattern. E_{θ} is now zero, which means that the intensity of the light is also zero. We have hed the first minimum, or dark fringe, in the diffraction pattern. The first and last ors now have a phase difference of 2π rad, wh tude E_{θ} is now zero, which means that the inte
reached the first minimum, or dark fringe, in th
phasors now have a phase difference of 2π rad
ference between the top and bottom rays the
Recall that this is the con hich means that the intensity of the light is also zero. We have
hum, or dark fringe, in the diffraction pattern. The first and last
ase difference of 2π rad, which means that the path length dif-
op and bottom rays th reached the first minimum, or dark fringe, in the diffraction pattern. The first and last phasors now have a phase difference of 2π rad, which means that the path length difference between the top and bottom rays throu Recall that this is the condition we determined for the first diffraction minimum.
 First Side Maximum. As we continue to increase θ , the angle $\Delta \phi$ between

adjacent phasors continues to increase, the chain of pha

sor.We have then reached the second minimum.

We could continue this qualitative method of determining the maxima and method.

Checkpoint 2

The figures represent, in smoother form (with more phasors) than Fig.36-7,the phasor diagrams for two points of a diffraction pattern that are on opposite sides of a certain diffraction maximum.(a) Which maximum is it? (b) What is the approximate value of m (in Eq. 36-3) that corresponds to this maximum?

Intensity in Single-Slit Diffraction, Quantitatively

Equation 36-3 tells us how to locate the minima of the single-slit diffraction pat-The figures represent, in smoother form (with more phasors)
than Fig. 36-7, the phasor diagrams for two points of a diffraction
pattern that are on opposite sides of a certain diffraction maxi-
mum. (a) Which maximum is i when that are on poposite sides of a certain diffraction maximum (a) Which maximum is it? (b) What is the approximate
when $\left(\frac{a}{u}\right)$ (*a*) when the pattern as a function shall prove below, that is maximum?
Intensity ntensity in Single-Slit Diffraction, Quantitatively

Equation 36-3 tells us how to locate the minima of the single-slit diffraction pat-

ern on screen C of Fig. 36-4 as a function of the angle θ in that figure. Here

$$
I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2, \tag{36-5}
$$

 $\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta.$ $\frac{\partial}{\partial t}$ sin θ .

The symbol α is just a convenient connection between the angle θ that locates a point on the viewing screen and the light intensity $I(\theta)$ at that point. The intensity I_m is the greatest value of the intensities $I(\theta)$ in the pattern and occurs at the central maximum (where $\theta = 0$), and ϕ is the phase difference (in radians) between $I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2$, (36-5)
 $\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta$. (36-6)

wenient connection between the angle θ that locates a

and the light intensity $I(\theta)$ at that point. The intensity

he intensities $I(\theta)$ in the pa $t(\theta) = I_m \left(\frac{-a}{\alpha} \right)$, (36-3)

Where $\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta$. (36-6)

The symbol α is just a convenient connection between the angle θ that locates a

point on the viewing screen and the light intensity $I(\theta)$ $\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta.$ (36-to

mient connection between the angle θ that locate

and the light intensity $I(\theta)$ at that point. The intensities $I(\theta)$ in the pattern and occurs at the c

intensities $I(\theta)$ in the $\sin \theta$. (36-6)

between the angle θ that locates a

sity $I(\theta)$ at that point. The intensity

1 the pattern and occurs at the cen-

see difference (in radians) between

1, 2, 3, (36-7)

1, 2, 3, (36-7) where $\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta$.

The symbol α is just a convenient connection between the angle θ that lcoint on the viewing screen and the light intensity $I(\theta)$ at that point. The ir I_m is the greatest value g screen and the light intensity $I(\theta)$ at that point. The of the intensities $I(\theta)$ in the pattern and occurre $\theta = 0$), and ϕ is the phase difference (in radia rays from the slit of width *a*.
5 shows that intensity ight intensity $I(\theta)$ at that point. The intensity
ies $I(\theta)$ in the pattern and occurs at the cen-
is the phase difference (in radians) between
of width *a*.
nsity minima will occur where
for $m = 1, 2, 3, ...$ (36-7)
find
f

$$
\alpha = m\pi, \qquad \text{for } m = 1, 2, 3, \dots \tag{36-7}
$$

$$
I_m
$$
 is the greatest value of the intensities $I(\theta)$ in the pattern and occurs at the central maximum (where $\theta = 0$), and ϕ is the phase difference (in radians) between the top and bottom rays from the slit of width *a*.
Study of Eq. 36-5 shows that intensity minima will occur where
 $\alpha = m\pi$, for $m = 1, 2, 3, ...$ (36-7)
If we put this result into Eq. 36-6, we find
 $m\pi = \frac{\pi a}{\lambda} \sin \theta$, for $m = 1, 2, 3, ...$,
or $a \sin \theta = m\lambda$, for $m = 1, 2, 3, ...$ (minima—dark fringes), (36-8)
which is exactly Eq. 36-3, the expression that we derived earlier for the location
of the minima.

which is exactly Eq. 36-3, the expression that we derived earlier for the location of the minima.

Plots. Figure 36-8 shows plots of the intensity of a single-slit diffraction pattern, **2008.** Supplementary and Se-2 INTENSIT and Se-2 INTENSIT \mathbf{P}_0 and Se-8 shows plots of the intensity of a single-slit diffraction position and Se-5 and 36-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and a Note that as calculated with Eqs. 36-5 and 36-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$. ¹⁰l. Note that as the slit width increases (relative to the wavelength), the width of the central diffraction maximum (the central hill-like region of the graphs) decreases; **Plots.** Figure 36-8 shows plots of the intensity of a single-slit diffraction pattern, calculated with Eqs. 36-5 and 36-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$. Note that as the slit width increases (rel **Plots.** Figure 36-8 shows plots of the intensity of a single-slit diffraction pattern,
calculated with Eqs. 36-5 and 36-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$.
Note that as the slit width increases (rel **Plots.** Figure 36-8 shows plots of the intensity of a single-slit diffraction pattern,
calculated with Eqs. 36-5 and 36-6 for three slit widths: $a = \lambda$, $a = 5\lambda$, and $a = 10\lambda$.
Note that as the slit width increases (rel longer have single-slit diffraction (but we still have diffraction due to the edges of $\frac{1}{20-15}$ the wide slit,like that produced by the edges of the razor blade in Fig.36-2). To find an expression for the intensity of a single-sint unitation pattern, we
the wavelets that a step sit width increases (relative to the wavelength), the width of the
central diffraction maximum (the central hill-like From the mass the simular micreases (elastive to the wavelength), the width of each at a stimular the central in Filiplic is the region of the graphs) decrease;
that is, the light undergoes less flaring by the slit. The s

Proof of Eqs. 36-5 and 36-6

need to divide the slit into many zones and then add the phasors corresponding to $\frac{1}{48}$ tential digitation matter interest in the ential fill-the ential digital of the viewing screen of Fig. 36-4, corre-
than wavelength A, the secondary maxima due to the slit disappear; we then no
the wide slit, like that pr sponding to a particular small angle θ . The amplitude E_{θ} of the resultant wave at P is
the vector sum of these phasors. If we divide the slit of Fig. 36-4 into infinitesimal The vector sum of these phasors. If we will not sum that of Fig. 36-4 into infinitesimal conducts and the slit of Fig. 36-2).
 Proof of Eqs. 36-5 and 36-6

To find an expression for the intensity at a point in the diffr Lotter that wavelets that reaction pursuant of the sum displayed, we have the solely of **Eqs.** 36-5 and 36-6
 Proof of Eqs. 36-5 and 36-6
 Proof of Eqs. 36-5 and 36-6
 Proof of Eqs. 36-5 and 36-6
 Proof of Eqs. 36compare interesting that as indicated by the edges of the razor blade in Fig. 36-2).
 Proof of Eqs. 36-5 and 36-6

To find an expression for the intensity at a point in the diffraction pattern, we

need to divide the sl **Proof of Eqs. 36-6** and produced by the edges of the razor biade in Fig. 30-2).
 Proof of Eqs. 36-5 and 36-6

To find an expression for the intensity at a point in the diffraction pattern, we

need to divide the slit i **Proof of Eqs. 36-5 and 36-6**

To find an expression for the intensity at a point in the diffraction pattern, we

need to divide the slit into many zones and then add the phasors corresponding to

those zones, as we did i The Lips. Jubid show the intensity at a point in the diffraction pattern, we

and an expression for the intensity at a point in the diffraction pattern, we

be zones, as we did in Fig. 36-7. The arc of phasors in Fig. 36-To find an expression for the intensity at a point in the diffraction pattern, we
need to divide the slit into many zones and then and the phasors corresponding to
the wavelets that reach an arbitrary point *P* on the vie need to divide the slit into many zones and then add the phasors corresponding to
those zones, as we did in Fig. 36-7. The arc of phasors in Fig. 36-9 represents
the wavelets that reach an arbitrary point P on the viewing those zones, as we did in Fig. 36-7. The arc of phasors in Fig. 36-9 represents
the wavelets that reach an arbitrary point *P* on the viewing screen of Fig. 36-4, corre-
sponding to a particular small angle θ . The ampl zones of width Δx , the arc of phasors in Fig. 36-9 approaches the arc of a circle; we

triangle we can write The angle ϕ in the lower part of Fig. 36-9 is the difference in phase between
the infinitesimal vectors at the left and right ends of arc E_m . From the geometry, ϕ
is also the angle between the two radii marked R i

$$
\sin\frac{1}{2}\phi = \frac{E_{\theta}}{2R}.
$$
\n(36-9)

In radian measure, ϕ is (with E_m considered to be a circular arc)

$$
\phi = \frac{E_m}{R}.
$$

Solving this equation for R and substituting in Eq. 36-9 lead to

$$
E_{\theta} = \frac{E_m}{\frac{1}{2}\phi} \sin \frac{1}{2}\phi. \tag{36-10}
$$

Intensity. In Module 33-2 we saw that the intensity of an electromagnetic Solving this equation for *R* and substituting in Eq. 36-9 lead to
 $E_{\theta} = \frac{E_m}{R}$.

Solving this equation for *R* and substituting in Eq. 36-9 lead to
 $E_{\theta} = \frac{E_m}{\frac{1}{2}\phi} \sin \frac{1}{2}\phi$.
 Intensity. In Module 33-2 we means that the maximum intensity I_m (at the center of the pattern) is proportional to E_m^2 and the intensity $I(\theta)$ at angle θ is proportional to E_{θ}^2 . Thus,
 $\frac{I(\theta)}{I} = \frac{E_{\theta}^2}{I^2}$. (36-11) Solving this equation for *K* and substituting in Eq. 36-9 lead to
 $E_{\theta} = \frac{E_m}{\frac{1}{2}\phi} \sin \frac{1}{2}\phi.$ (36-10) centra
 Intensity. In Module 33-2 we saw that the intensity of an electromagnetic

wave is proportional to t

$$
\frac{I(\theta)}{I_m} = \frac{E_{\theta}^2}{E_m^2}.
$$
\n(36-11)

\nand then substituting $\alpha = \frac{1}{2}\phi$, we are led to Eq.

\n(36-12)

\n(36-13)

\n(36-14)

\n(36-14)

\n(36-15)

\n(36-16)

\n(36-17)

36-5 for the intensity as a function of θ :

$$
I(\theta)=I_m\bigg(\frac{\sin\alpha}{\alpha}\bigg)^2.
$$

between the rays from the top and bottom of the entire slit may be related to a bonal to E_{m}^{*} and the intensity $I(\theta)$ at angle θ is proportional to E_{θ}^{*} . Thus,
 $\frac{I(\theta)}{I_{m}} = \frac{E_{\theta}^{2}}{E_{m}^{2}}$. (36

Substituting for E_{θ} with Eq. 36-10 and then substituting $\alpha = \frac{1}{2}\phi$, we are Substituting for E_a with Eq. 36-10 and then substituting $\alpha = \frac{1}{2}\phi$, we are led to Eq.

36-5 for the intensity as a function of θ :
 $I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2$.

The second equation we wish to prove relates α to 36-5 for the intensity as a function of θ :
 $I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2$.

The second equation we wish to prove relates α to θ . The phase difference ϕ

between the rays from the top and bottom of the entire slit

$$
\phi = \left(\frac{2\pi}{\lambda}\right)(a\sin\theta),
$$

(36-10) central diffraction maximum. Figure 36-8 The relative intensity in single-slit

Figure 36-9 A construction used to calculate the intensity in single-slit diffraction.The situation shown corresponds to that of

are used separately in a single-slit diffraction experiment.The figure shows the results as graphs of intensity I versus angle θ for the two dif-**Checkpoint 3**
Two wavelengths, 650 and 430 nm,
are used separately in a single-slit dif-
fraction experiment. The figure
shows the results as graphs of inten-
sity *I* versus angle θ for the two dif-
fraction patterns color will be seen in the combined diffraction pattern at (a) angle A

Sample Problem 36.02 Intensities of the maxima in a single-slit interference pattern
Find the intensities of the first three secondary maxima intensities at those maxima, we get
(side maxima) in the single-slit diffract Find the intensities of the first three secondary maxima Fraction patterns. If both wavelengths

are then used simultaneously, what

color will be seen in the combined

diffraction pattern at (a) angle A

and (b) angle B?

Sample Problem 36.02 Intensities of the maxima in a sin measured as a percentage of the intensity of the central maximum.

KEY IDEAS

The secondary maxima lie approximately halfway between **Sample Problem 36.02 Intensities of the maxima in a single-slit interference** [

Find the intensities of the first three secondary maxima intensities at those maxima, we g

(side maxima) in the single-slit diffraction pa $(\alpha = m\pi)$. The locations of the secondary maxima are then **uple Problem 36.02 Intensities of the maxima in a single-slit**
the intensities of the first three secondary maxima intensities at thos
maxima) in the single-slit diffraction pattern of Fig. 36-1,
ured as a percentage of given (approximately) by measured as a percentage of the intensity of the central $\frac{I}{I_m} = \begin{pmatrix} \text{maximum.} & \frac{I}{I_m} \\ \text{The first} & \text{relative} \end{pmatrix}$

The secondary maxima lie approximately halfway between

the minima, whose angular locations are given by Eq. 36 **EX IDEAS**

The secondary maxima lie approximately halfway between

The first of the second

relative intensity is

The minima, whose angular locations are given by Eq. 36-7
 $(\alpha = m\pi)$. The locations of the secondary maxi **The secondary maxima lie approximately halfway between**
the minima, whose angular locations are given by Eq. 36-7
($\alpha = m\pi$). The locations of the secondary maxima are then
given (approximately) by
 $a = (m + \frac{1}{2})\pi$, for The secondary maxima lie approximately halfway between

the minima, whose angular locations are given by Eq. 36-7 $\frac{I_1}{I_m} =$
 $(\alpha = m\pi)$. The locations of the secondary maxima are then

given (approximately) by
 $a = (m + \$

$$
a = (m + \frac{1}{2})\pi
$$
, for $m = 1, 2, 3, ...$,

$$
\frac{I}{I_m} = \left(\frac{\sin \alpha}{\alpha}\right)^2 = \left(\frac{\sin((m + \frac{1}{2})\pi)}{(m + \frac{1}{2})\pi}\right)^2, \text{ for } m = 1, 2, 3, \dots
$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

The first of the secondary maxima occurs for
$$
m = 1
$$
, and its
\nrelative intensity is
\n
$$
\frac{I_1}{I_m} = \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2
$$
\n
$$
= 4.50 \times 10^{-2} \approx 4.5\%.
$$
 (Answer)
\nFor $m = 2$ and $m = 3$ we find that
\n
$$
\frac{I_2}{I_m} = 1.6\% \text{ and } \frac{I_3}{I_m} = 0.83\%.
$$
 (Answer)
\nAs you can see from these results, successive secondary
\nmaxima decrease rapidly in intensity. Figure 36-1 was deliberately overexposed to reveal them.

 $1, 2, 3, \ldots$, For $m = 2$ and $m = 3$ we find that

$$
= 4.50 \times 10^{-2} \approx 4.5\%.
$$
 (Answer)
and $m = 3$ we find that

$$
\frac{I_2}{I_m} = 1.6\% \text{ and } \frac{I_3}{I_m} = 0.83\%.
$$
 (Answer)

erately overexposed to reveal them. **Calculations:** Substituting the approximate values of α for maxima decrease rapidly in intensity. Figure 36-1 was delib-

PLUS Additional examples, video, and practice available at WileyPLUS

36-3 DIFFRACTION BY A CIRCULAR APERTURE

Learning Objectives

After reading this module, you should be able to . . .

- small circular aperture or obstacle.
- 36.14 For diffraction by a small circular aperture or obstacle, apply the relationships between the angle θ to the first minimum, the wavelength λ of the light, the diameter d of the aperture, the distance D to a viewing screen, and the distance y between the minimum and the center of the diffraction pattern.
- **36.15** By discussing the diffraction patterns of point objects,

36.13 Describe and sketch the diffraction pattern from a explain how diffraction limits visual resolution of objects.

- explain how diffraction limits visual resolution of objects.
 In the following the computation of the following the following the followith of the following the following the following the following proximate) angle at wh 36.16 Identify that Rayleigh's criterion for resolvability gives the (approximate) angle at which two point objects are just barely resolvable.
- **36.17** Apply the relationships between the angle θ_R in Rayleigh's criterion, the wavelength λ of the light, the diameter d of the aperture (for example, the diameter of **Explain how diffraction limits visual resolution of objects.**
 16 Identify that Rayleigh's criterion for resolvability gives

the (approximate) angle at which two point objects are just

barely resolvable.
 17 Apply point objects, and the distance L to those objects.

can then be no less than

light passes.

verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation

in which d is the diameter of the aperture through which the

 $\theta_{\rm R} = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion),

Key Ideas

● Diffraction by a circular aperture or a lens with diameter d produces a central maximum and concentric maxima and minima, with the first minimum at an angle θ given by 36-3 DIFFRACTION

r aperture or a lens with

entral maximum and concentric is at the first minimum at an angle θ

(first minimum—circular aperture).

(first minimum—circular aperture).

in which *d* is the

light passe

$$
\sin \theta = 1.22 \frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}.
$$

● Rayleigh's criterion suggests that two objects are on the

Diffraction by a Circular Aperture

maxima and minima, with the first minimum at an angle θ can then be no less than
maxima and minima, with the first minimum at an angle θ can then be no less than
given by
 $\sin \theta = 1.22 \frac{\lambda}{d}$ (first minimum—circular such as a circular lens, through which light can pass. Figure 36-10 shows the image of the aperture.
 • Rayleigh's criterion suggests that two objects are on the input in which *d* is the diameter of the aperture in the formed by light from a laser that was directed onto a circular aperture with a very sin $\theta = 1.22 \frac{\lambda}{d}$ (first minimum—circular aperture).
 • Rayleigh's criterion suggests that two objects are on the light passes.
 Diffraction by a Circular Aperture
 Figure 2018 and the substrate of the aperture a circular disk surrounded by several progressively fainter secondary rings. **• Rayleigh's criterion suggests that two objects are on the** light passes.
 Diffraction by a Circular Aperture

Here we consider diffraction by a circular aperture—that is, a circular opening,

such as a circular lens, **Diffraction by a Circular Aperture**

Here we consider diffraction by a circular aperture—that is, a circular opening,

such as a circular lens, through which light can pass. Figure 36-10 shows the image

formed by light than a rectangular slit. and the central axis to any point on that (circular)

and the central axis to any point, as geometrical optics would suggest, but

several progressively fainter secondary rings.

Suittle doubt that we are dealing with a d a circular disk surrounded by several progressively fainter secondary
a circular disk surrounded by several progressively fainter secondary
Comparison with Fig. 36-1 leaves little doubt that we are dealing with a tion phe (first minimum—single slit), and a circle of the circular shows that the first minimum for the entrained as is located by $\frac{1}{2}$ figure 36-10 The courtesy Jean aperture of diameter *d* is located by $\frac{1}{2}$ figure $\$

The (complex) analysis of such patterns shows that the first minimum for the diffraction pattern of a circular aperture of diameter d is located by
cular aperture. Note the central maximum

$$
\sin \theta = 1.22 \frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}.\tag{36-12}
$$

The angle θ here is the angle from the central axis to any point on that (circular)

$$
\sin \theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit)}, \tag{36-12}
$$

which locates the first minimum for a long narrow slit of width a . The main differ-Figure 3

diffraction pattern of a circular aperture of diameter d is located by

sin $\theta = 1.22 \frac{\lambda}{d}$ (first minimum—circular aperture). (36-12)

The angle θ here is the angle from the central axis to any point on th

Resolvability

The fact that lens images are diffraction patterns is important when we wish to resolve (distinguish) two distant point objects whose angular separation is small. The angle θ here is the angle from the central axis to any point on that (circular)
minimum. Compare this with Eq. 36-1,
sin $\theta = \frac{\lambda}{a}$ (first minimum—single slit), (36-13)
which locates the first minimum for a long minimum. Compare this with Eq. 36-1,
 $\sin \theta = \frac{\lambda}{a}$ (first minimum – single slit), (36-13)

which locates the first minimum for a long narrow slit of width a. The main difference

is the factor 1.22, which enters because **Resolvability**
The fact that lens images are diffraction patterns is importaneously (distinguish) two distant point objects whose angula
Figure 36-11 shows, in three different cases, the visual apponding intensity patter

Courtesy Jearl Walker
 Figure 36-10 The diffraction pattern of a cir-

cular aperture. Note the central maximum

and the circular secondary maxima. The

figure has been overexposed to bring out

these secondary maxima, w Courtesy Jearl Walker
 Figure 36-10 The diffraction pattern of a cir-

cular aperture. Note the central maximum

and the circular secondary maxima. The

figure has been overexposed to bring out

these secondary maxima, w

Courtesy Jearl Walker

Figure 36-10 The diffraction pattern of a cirfigure has been overexposed to bring out Courtesy Jearl Walker
 Figure 36-10 The diffraction pattern of a cir-

cular aperture. Note the central maximum

and the circular secondary maxima. The

figure has been overexposed to bring out

these secondary maxima, w less intense than the central maximum.

Figure 36-11 At the top, the images of
two point sources (stars) formed by a
converging lens. At the bottom, repre-(a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished.Rayleigh's crite-Figure 36-11 At the top, the images of
two point sources (stars) formed by a
converging lens. At the bottom, repre-
sentations of the image intensities. In
(a) the angular separation of the
sources is too small for them t maximum of one diffraction pattern coinciding with the first minimum of

angular separation. In Figure 36-11*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished f angular separation. In Figure 36-11*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished f so much that is, their diffraction patterns (mainly their central maxima) overlap
so much that the two objects cannot be distinguished from a single point object. In
Fig. 36-11*b* the objects are barely resolved, and in Fi Solution: In Figure 36-11*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished from a singl In Fig. 36-11b The angular separation. In Figure 36-11*a*, the objects are not resolved because of action; that is, their diffraction patterns (mainly their central maxima) overlap unch that the two objects cannot be disti

central maximum of the diffraction pattern of one source is centered on the first angular separation. In Figure 36-11*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects cannot be distinguished f angular separation. In Figure 36-11*a*, the objects are not resolved because of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap so much that the two objects ran barely resolved, and by this criterion must have an angular separation θ_R of diffraction; that is, their diffraction patterns (mainly their central maxima) overlap
so much that the two objects are barely resolved, and in Fig. 36-11*b* the objects are barely resolved, and in Fig. 36-11*b* the angul

$$
\theta_{\rm R} = \sin^{-1} \frac{1.22\lambda}{d}.
$$

$$
\theta_{\rm R} = 1.22 \frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}.
$$
\n(36-14)

Human Vision. Applying Rayleigh's criterion for resolvability to human vision is only an approximation because visual resolvability depends on many facby this criterion must have an angular separation θ_R of
 $\theta_R = \sin^{-1} \frac{1.22\lambda}{d}$.

Since the angles are small, we can replace sin θ_R with θ_R expressed in radians:
 $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion). (36-1 $\theta_R = \sin^{-1} \frac{1.22\lambda}{d}$.

Since the angles are small, we can replace $\sin \theta_R$ with θ_R expressed in radians:
 $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion). (36-14)
 Human Vision. Applying Rayleigh's criterion for resolvab $\theta_R = \sin^{-1} \frac{2\pi \mu_{\text{tot}}}{d}$.

Since the angles are small, we can replace $\sin \theta_R$ with θ_R expressed in radians:
 $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion). (36-14)
 Human Vision. Applying Rayleigh's criterion for res tion that can actually be resolved by a person is generally somewhat greater than Since the angles are small, we can replace sin θ_R with θ_R expressed in radians:
 $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion). (36-14)
 Human Vision. Applying Rayleigh's criterion for resolvability to human

vision i **Human Vision.** Applying Rayleigh's criterion). (36-14)
 Human Vision. Applying Rayleigh's criterion for resolvability to human

vision is only an approximation because visual resolvability depends on many fac-

tors, $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion). (36-14)
 Human Vision. Applying Rayleigh's criterion for resolvability to human

vision is only an approximation because visual resolvability depends on many fac-

tors, such a **Human Vision.** Applying Rayleigh's criterion for resolvability to human vision is only an approximation because visual resolvability depends on many factors, such as the relative brightness of the sources and their surro vision is only an approximation because visual resolvability depends on many fac-
tors, such as the relative brightness of the sources and their surroundings, turbu-
lence in the in between the sources and the observer, a

Pointillism. Rayleigh's criterion can explain the arresting illusions of painting is made not with brush strokes in the usual sense but rather with a tors, such as the relative brightness of the sources and their surroundings, turbu-
lence in the air between the sources and the observer, and the functioning of the
observer's visual system. Experimental results show tha lence in the air between the sources and the observer, and the functioning of the observer's visual system. Experimental results show that the least angular separation that can actually be resolved by a person is generall observer's visual system. Experimental results show that the least angular separa-
tion that can actually be resolved by a person is generally somewhat greater than
the value given by Eq. 36-14. However, for calculations tion that can actually be resolved by a person is generally somewhat greater than the value given by Eq. 36-14. However, for calculations here, we shall take Eq. 36-14 greater than θ_R , we can visually resolve the sourc the value given by Eq. 36-14. However, for calculations here, we shall take Eq. 36-
14 as being a precise criterion: If the angular separation θ between the sources is
greater than θ_R , we can visually resolve the so

Figure 36-12 The pointillistic painting The Seine at Herblay by Maximilien Luce consists of thousands of colored dots.With the and their true colors are visible.At normal Figure 36-12 The pointillistic painting *The*
Seine at Herblay by Maximilien Luce consists of thousands of colored dots. With the
viewer very close to the canvas, the dots
and their true colors are visible. At normal
view and thus blend.

Maximilien Luce, The Seine at Herblay, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

36-3 DIFFRACTION BY A CIRCULAR A
you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors
"make un" a color for that 36-3 DIFFRACTION BY A CIRCUL
you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors
coming into your eye from any g coming into your eye from any group of dots can then cause your brain to 36-3 DIFFRACTION BY A CIRCULAR

you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors

coming into your eye from 36-3 DIFFRACTION BY A CIRCULAR
you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors
coming into your eye from any colors of the art. 36-3 DIFFRACTION BY A CIRCUL

you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors

coming into your eye from an 36-3 DIFFRACTION BY A CIRCUL
you stand far enough from the painting, the angular separations θ are less than
 θ_R and the dots cannot be seen individually. The resulting blend of colors
coming into your eye from any g

When we wish to use a lens instead of our visual system to resolve objects of you stand far enough from the painting, the angular separations θ are less than θ_R and the dots cannot be seen individually. The resulting blend of colors coming into your eye from any group of dots can then cause y light is often used with microscopes because its wavelength is shorter than a visible light wavelength. but the art.

When we wish to use a lens instead of our visual system to resolve objects of

saisible. According to Eq. 36-14, this can be done either by increasing the lens

sameter or by using light of a shorter waveleng When we wish to use a lens instead of our visual system to resolve objects of nall angular separation, it is desirable to make the diffraction pattern as small as sssible. According to Eq. 36-14, this can be done either by nall angular separation, it is desirable to make the diffraction pattern as small as
ssible. According to Eq. 36-14, this can be done either by increasing the lens
ameter or by using light of a shorter wavelength. For this

Checkpoint 4

Suppose that you can barely resolve two red dots because of diffraction by the red dots because of diffraction by the

al illumination around you so that the

blvability of the dots improve or diminish?

beriment to check your answer.)
 tillistic paintings use the diffraction of you

tillistic **pa** The distinguish and solution around you so that the

1.5 mm and

Sample Problem 36.03 Pointillistic paintings use the diffraction of your eye Figure 36-13*a* is a representation of the colored dots on a Rayleigh's criterion: pointillistic painting. Assume that the average center-Figure 36-13a is a representation of the colored dots on a **Checkpoint 4**

Suppose that you can barely resolve two red dots because of diffracti

pupil of your eye. If we increase the general illumination around you

pupil decreases in diameter, does the resolvability of the dots to-center separation of the dots is $D = 2.0$ mm. Also assume that the diameter of the pupil of your eye is $d = 1.5$ mm and that the least angular separation between dots you can **Calculations:** Figure 36-13b shows, from the side, the **Example Problem 36.03 Pointillistic paintings use the diffraction of**

Figure 36-13*a* is a representation of the colored dots on a

Figure 36-13*a* is a representation of the colored dots on a

Pointillistic painting. A viewing distance from which you cannot distinguish any separation D , and your distance L from them. Because dots on the painting? pointillistic painting. Assume that the average center-
to-center separation of the dots is $D = 2.0$ mm. Also assume
that the diameter of the pupil of your eye is $d = 1.5$ mm and
that the least angular separation between

KEY IDEA

Consider any two adjacent dots that you can distinguish continue to distinguish the dots until their angular separacontinue to distinguish the dots until their angular separa-
tion θ (in your view) has decreased to the angle given by

Figure 36-13 (a) Representation of some dots on a pointillistic paint-

Rayleigh's criterion:

$$
\theta_{\rm R} = 1.22 \frac{\lambda}{d}.\tag{36-15}
$$

Calculations: Figure 36-13*b* shows, from the side, the angular separation *b* of the dots, their center-to-center separation *D*, and your distance *L* from them. Because D/L is small angle θ is also small and we c **the diffraction of your eye**

Rayleigh's criterion:
 $\theta_R = 1.22 \frac{\lambda}{d}$. (36-15)
 Calculations: Figure 36-13b shows, from the side, the

angular separation θ of the dots, their center-to-center

separation D, and yo **Solution Standard Exercise 2.133**
 Standard Standard Properties Algebra 2.133
 Calculations: Figure 36-13*b* shows, from the side, the

angular separation θ of the dots, their center-to-center

separation *D*, and **Calculations:** Equivariantly $\theta_R = 1.22 \frac{\lambda}{d}$. (36-15)
 Calculations: Figure 36-13*b* shows, from the side, the angular separation θ of the dots, their center-to-center separation *D*, and your distance *L* from t approximation $\theta_R = 1.22 \frac{\lambda}{d}$. (36-15)

culations: Figure 36-13*b* shows, from the side, the

lar separation θ of the dots, their center-to-center

ration *D*, and your distance *L* from them. Because

is small, angle θ is als $\theta_R = 1.22 \frac{d}{d}$.
 Calculations: Figure 36-13*b* shows, from the

angular separation θ of the dots, their center-

separation *D*, and your distance *L* from them.
 D/L is small, angle θ is also small and we c

$$
\theta = \frac{D}{L}.\tag{36-16}
$$

$$
L = \frac{Dd}{1.22\lambda}.\tag{36-17}
$$

separation *D*, and your distance *L* from them. Because
 D/L is small, angle θ is also small and we can make the

approximation
 $\theta = \frac{D}{L}$. (36-16)

Setting θ of Eq. 36-16 equal to θ_R of Eq. 36-15 and solv-
 D/L is small, angle θ is also small and we can make the
approximation
 $\theta = \frac{D}{L}$. (36-16)
Setting θ of Eq. 36-16 equal to θ_R of Eq. 36-15 and solv-
ing for *L*, we then have
 $L = \frac{Dd}{1.22\lambda}$. (36-17)
Equatio wavelengths) become indistinguishable before adjacent blue dots do. To find the least distance L at which no colored $\theta = \frac{D}{L}$. (36-16)

Setting θ of Eq. 36-16 equal to θ_R of Eq. 36-15 and solv-

ing for *L*, we then have
 $L = \frac{Dd}{1.22\lambda}$. (36-17)

Equation 36-17 tells us that *L* is larger for smaller λ . Thus, as

you mov (36-16)
 $.36-15$ and solv-
 $(36-17)$

maller λ . Thus, as

it red dots (long

before adjacent

which *no* colored

400 nm (blue or Setting θ of Eq. 36-16 equal to θ_R of Eq. 36-15 and solv-
ing for *L*, we then have
 $L = \frac{Dd}{1.22\lambda}$. (36-17)
Equation 36-17 tells us that *L* is larger for smaller λ . Thus, as
you move away from the painting, a At this or a greater distance, the color you perceive at
given spottent and the painting, adjacent red dots (long
elengths) become indistinguishable before adjacent
dots do. To find the least distance L at which no color dots are distinguishable, we substitute $\lambda = 400$ nm (blue or

$$
L = \frac{(2.0 \times 10^{-3} \text{ m})(1.5 \times 10^{-3} \text{ m})}{(1.22)(400 \times 10^{-9} \text{ m})} = 6.1 \text{ m. (Answer)}
$$

any given spot on the painting is a blended color that may not actually exist there.

Sample Problem 36.04 Rayleigh's criterion for resolving two distant objects

1094 CHAPTER 36 DIFFRACTION
 Sample Problem 36.04 Rayleigh's criterion for resolving two distant o

A circular converging lens, with diameter $d = 32$ mm and focal

length $f = 24$ cm, forms images of distant point obje length $f = 24$ cm, forms images of distant point objects in the fo-CHAPTER 36 DIFFRACTION

24 **Problem 36.04 Rayleigh's criterion for**

24 cm, forms images of distant point objects in the fo-

24 cm, forms images of distant point objects in the fo-

14 f the lens. The wavelength is $\lambda =$ cal plane of the lens. The wavelength is $\lambda = 550$ nm. A circular converging lens, with diameter $d = 32$ mm and focal

**Sample Problem 36.04 Rayleigh's criterion for resolving two dis

A circular converging lens, with diameter** $d = 32$ **mm and focal

length** $f = 24$ **cm, forms images of distant point objects in the fo-

cal plane of the lens.** tion must two distant point objects have to satisfy Rayleigh's criterion?

KEY IDEA

Figure 36-14 shows two distant point objects P_1 and P_2 , the through a co Figure 36-14 shows two distant point objects in the fo-

length $f = 24$ cm, forms images of distant point objects in the fo-

cal plane of the lens. The wavelength is $\lambda = 550$ nm.

(a) Considering diffraction by the lens position on the screen for the central maxima of the images (a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh's criterion?
 KEY IDEA

Figure 36-14 shows two distant point objects P_1 and P_2 , the through formed by the lens. Note that the angular separation θ_0 of the objects equals the angular separation θ_i of the images. **Thus, if the images are to satisfy Rayleigh's criterion?**
 The image SE-14 Light from two distant point objects P_1 and P_2 , the **Figure 36-14** Light from two distant performed be images and a viewing screen in the **KEY IDEA**
 KEY IDEA
 Figure 36-14 shows two distant point objects P_1 and P_2 , the through a lens, and a viewing screen in the focal plane of the lens. It the focal palso shows, on the right, plots of light inten **KEY IDEA**

Figure 36-14 shows two distant point objects P_1 and P_2 , the

lens, and a viewing screen in the focal plane of the lens. It

also shows, on the right, plots of light intensity I versus

position on the sc

$$
\theta_o = \theta_i = \theta_R = 1.22 \frac{\lambda}{d}
$$
\nthe screen
\nRearranging
\n
$$
= \frac{(1.22)(550 \times 10^{-9} \text{ m})}{32 \times 10^{-3} \text{ m}} = 2.1 \times 10^{-5} \text{ rad. (Answer)}
$$
\n
$$
\tan \theta \approx \theta,
$$

Each central maximum in the two intensity curves of Fig. When $= \frac{(2.22)(8.6 \times 10^{-3} \text{ m})}{32 \times 10^{-3} \text{ m}} = 2.1 \times 10^{-5} \text{ rad.}$ (Answer)
Each central maximum in the two intensity curves of Fig.

Figure 36-14 Light from two distant point objects P_1 and P_2 passes through a converging lens and forms images on a viewing screen in object is shown.The images are not points but diffraction patterns, with intensities approximately as plotted at the right. Figure 36-14 Light from two distant point objects P_1 and P_2 passes
through a converging lens and forms images on a viewing screen in
the focal plane of the lens. Only one representative ray from each
object is shown Hymer 36-14 Light from two distant point objects P_1 and P_2 passes
through a converging lens and forms images on a viewing screen in
the focal plane of the lens. Only one representative ray from each
object is shown. **Example 36-44** Light from two distant point objects P_1 and P_2 passes
through a converging lens and forms images on a viewing screen in
the focal plane of the lens. Only one representative ray from each
object is sh

(b) What is the separation Δx of the centers of the *images* in tral peaks in the two intensity-versus-position curves?)

Calculations: From either triangle between the lens and the screen in Fig. 36-14, we see that tan $\theta_i/2 = \Delta x/2f$. the local plane of the lens. Only one representative ray in
object is shown. The images are not points but diffraction
with intensities approximately as plotted at the right.
(b) What is the separation Δx of the center otted at the right.
 \therefore (the centers of the *images* in

is the separation of the *cen*-

rsus-position curves?)

ngle between the lens and

see that tan $\theta_i/2 = \Delta x/2f$.

making the approximation

, (36-18)

e then fin is the separation Δx of the centers of the *imag*

I plane? (That is, what is the separation of the

s in the two intensity-versus-position curves?)
 ions: From either triangle between the lens

een in Fig. 36-14, centers of the *images* in

e separation of the *cen*-

position curves?)

between the lens and

that tan $\theta_i/2 = \Delta x/2f$.

sing the approximation

(36-18)

n find

5.0 μ m. (Answer)

$$
\Delta x = f\theta_i,\tag{36-18}
$$

where θ_i is in radian measure. We then find

$$
\Delta x = (0.24 \text{ m})(2.1 \times 10^{-5} \text{ rad}) = 5.0 \text{ }\mu\text{m}.
$$
 (Answer)

PLUS Additional examples, video, and practice available at WileyPLUS

DIFFRACTION BY A DOUBLE SLIT

Learning Objectives

After reading this module, you should be able to ...

36.18 In a sketch of a double-slit experiment, explain **EVS**
 **Additional examples, video, and practice available

6** – **4** DIFFRACTION BY A DOUBLE SLIT

arning Objectives

For reading this module, you should be able to ...

18 In a sketch of a double-slit experiment, explain interference pattern, and identify the diffraction envelope, the central peak, and the side peaks of that envelope.

36.19 For a given point in a double-slit diffraction pattern, center of the pattern.

36.20 In the intensity equation for a double-slit diffraction

Key Ideas

● Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.

 \bullet For identical slits with width a and center-to-center separation d , the intensity in the pattern varies with the angle θ from the central axis as

$$
I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad \text{(double slit)},
$$

pattern, identify what part corresponds to the interference between the two slits and what part corresponds to the diffraction by each slit.

calculate the intensity I in terms of the intensity I_m at the single-slit diffraction pattern, and then count the number of center of the pattern.

two-slit maxima that are contained in the central peak and in 36.21 For double-slit diffraction, apply the relationship between the ratio d/a and the locations of the diffraction minima in the pattern, identify what part corresponds to the interference
between the two slits and what part corresponds to the
diffraction by each slit.
.21 For double-slit diffraction, apply the relationship between
the ratio d/a pattern, identify what part corresponds to the interference
between the two slits and what part corresponds to the
diffraction by each slit.
21 For double-slit diffraction, apply the relationship between
the ratio d/a the side peaks of the diffraction envelope.

where I_m is the intensity at the center of the pattern,

$$
\beta = \left(\frac{\pi d}{\lambda}\right) \sin \theta,
$$

and
$$
\alpha = \left(\frac{\pi a}{\lambda}\right) \sin \theta.
$$

Diffraction by a Double Slit

Diffraction by a Double Slit

In the double-slit experiments of Chapter 35, we implicitly assumed that the slits

were much narrower than the wavelength of the light illuminating them; that is,
 $a \ll \lambda$. For such narrow **Diffraction by a Double Slit**
In the double-slit experiments of Chapter 35, we implicitly assumed that the slits
were much narrower than the wavelength of the light illuminating them; that is,
 $a \ll \lambda$. For such narrow sl **Diffraction by a Double Slit**
In the double-slit experiments of Chapter 35, we implicitly assumed that the slits
were much narrower than the wavelength of the light illuminating them; that is,
 $a \ll \lambda$. For such narrow sl from the two slits produces bright fringes with approximately the same intensity **Diffraction by a Double Slit**
In the double-slit experiments of Chapter 35,
were much narrower than the wavelength of
 $a \ll \lambda$. For such narrow slits, the central max
either slit covers the entire viewing screen. I
from t **Taction by a Double Slit**

and double-slit experiments of Chapter 35, we implicitly assumed that the slits

be much narrower than the wavelength of the light illuminating them; that is,
 λ . For such narrow slits, the **Diffraction by a Double Slit**
In the double-slit experiments of Chapter 35, we implicitly assumed that the slits
were much narrower than the wavelength of the light illuminating them; that is,
 $a \ll \lambda$. For such narrow sl **Diffraction by a Double Slit**
In the double-slit experiments of Chapter 35, we implicitly assumed that the slits
were much narrower than the wavelength of the light illuminating them; that is,
 $a \ll \lambda$. For such narrow sl

fringes produced by double-slit interference (as discussed in Chapter 35) are modified by diffraction of the light passing through each slit (as discussed in this chapter). e much narrower than the wavelength of the light illuminating them; that is, $λ$. For such narrow slits, the central maximum of the diffraction pattern of er slit covers the entire viewing screen. Moreover, the interfere either slit covers the entire viewing screen. Moreover, the interference of light
from the two slits produces bright fringes with approximately the same intensity
(Fig. 35-12).
In practice with visible light, however, the

slit interference pattern that would occur if the slits were infinitely narrow (and from the two slits produces bright fringes with approximately the same intensity (Fig. 35-12).

In practice with visible light, however, the condition $a \ll \lambda$ is often not met.

For relatively wide slits, the interference fraction pattern has a broad central maximum and weaker secondary maxima at In practice with visible light, however, the condition $a \ll \lambda$ is often not met.
For relatively wide slits, the interference of light from two slits produces bright
fringes that do not all have the same intensity. That is For relatively wide slits, the interference of light from two slits produces bright fringes that do not all have the same intensity. That is, the intensities of the migres produced by double-slit interference (as discusse fringes that do not all have the same intensity. That is, the intensities of the fringes produced by double-slit interference (as discussed in Chapter 35) are modified by durifraction of the light passing through each sli intensities are affected.

ble-slit interference experiment with vanishingly narrow **Figure 36-15** (*a*) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (*b*) The intensity plot for diffraction by a typical slit of width *a* (not vanishingly narro **Figure 36-15** (a) The intensity plot to be expected in a dou-
ble-slit interference experiment with vanishingly narrow
slits. (b) The intensity plot for diffraction by a typical slit
of width a (not vanishingly narrow). plot to be expected for two slits of width a.The curve of (b) acts as an envelope, limiting the intensity of the dou-**Figure 36-15** (*a*). The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (*b*). The intensity plot for diffraction by a typical slit of width *a* (not vanishingly nar diffraction pattern of (b) eliminate the double-slit **Figure 36-15** (*a*) The intensity plot to be expected in a dou-
ble-slit interference experiment with vanishingly narrow
slits. (*b*) The intensity plot for diffraction by a typical slit
of width *a* (not vanishingly nar

Figure 36-16 (*a*) Interference fringes for an

Courtesy Jearl Walker

Photos. Figure 36-16*a* shows an actual pattern in which both double-slit Courtesy Jearl Walker
 Photos. Figure 36-16a shows an actual pattern in which both double-slit

interference and diffraction are evident. If one slit is covered, the single-slit

diffraction pattern of Fig. 36-16b resul Courtesy Jearl Walker
 Photos. Figure 36-16*a* shows an actual pattern in which both double-slit

interference and diffraction are evident. If one slit is covered, the single-slit

diffraction pattern of Fig. 36-16*b* re bring out the faint secondary maxima and that several secondary maxima (rather than one) are shown. **Photos.** Figure 36-16*a* shows an actual pattern in which both double-slit reference and diffraction are evident. If one slit is covered, the single-slit raction pattern of Fig. 36-16*b* results. Note the correspondence Eigs. 36-16a and 36-15c, and between Figs. 36-16b and 36-15c, in component elective the correspondence between

Figs. 36-16a and 36-15c, and between Figs. 36-16b and 36-15b. In comparing

these figures, bear in mind that

slit interference pattern is given by

bring out the faint secondary maxima and that several secondary maxima (rather than one) are shown.

\nIntensity. With diffraction effects taken into account, the intensity of a double-slit interference pattern is given by

\n
$$
I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha} \right)^2 \quad \text{(double-slit)}, \tag{36-19}
$$
\nin which

\n
$$
\beta = \frac{\pi d}{\lambda} \sin \theta \quad \text{(36-20)}
$$
\nand

\n
$$
\alpha = \frac{\pi a}{\lambda} \sin \theta. \tag{36-21}
$$
\nHere *d* is the distance between the centers of the slits and *a* is the slit width. Note carefully that the right side of Eq. 36-19 is the product of *I*, and two factors (1) The

 πd . α $\frac{\partial u}{\partial x}$ sin θ

 $\alpha = \frac{\pi a}{\lambda} \sin \theta$. $\beta = \frac{\pi a}{\lambda} \sin \theta$
 $\alpha = \frac{\pi a}{\lambda} \sin \theta$.

Intensity. With diffraction effects taken into account, the intensity of a double-
slit interference pattern is given by
 $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2$ (double slit), (36-19)
in which $\beta = \frac{\pi d}{\lambda} \sin \theta$ (36-20)
and α slit interference pattern is given by
 $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2$ (double slit), (36-19)

in which $\beta = \frac{\pi d}{\lambda} \sin \theta$ (36-20)

and $\alpha = \frac{\pi a}{\lambda} \sin \theta$. (36-21)

Here *d* is the distance between the centers of the slit *interference factor* $\cos^2 \beta$ is due to the interference between two slits with slit separa- $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)$ (double slit), (36-19)

in which $\beta = \frac{\pi d}{\lambda} \sin \theta$ (36-20)

and $\alpha = \frac{\pi a}{\lambda} \sin \theta$. (36-21)

Here *d* is the distance between the centers of the slits and *a* is the slit width. Note

caref tion d (as given by Eqs. 35-22 and 35-23). (2) The diffraction factor $[(\sin \alpha)/\alpha]^2$ is due to diffraction by a single slit of width a (as given by Eqs. 36-5 and 36-6). In which $β = \frac{πd}{λ} \sin θ$ (36-20)

and $α = \frac{πa}{λ} \sin θ$. (36-21)

Here *d* is the distance between the cneters of the slits and *a* is the slit width. Note

carefully that the right side of Eq. 36-19 is the product of

describing the interference pattern for a pair of vanishingly narrow slits with slit and
 $\alpha = \frac{\pi a}{\lambda} \sin \theta$.

Here *d* is the distance between the centers of the slits and *a* is th

carefully that the right side of Eq. 36-19 is the product of I_m and tv
 interference factor cos² β is due to the = $\frac{\pi a}{\lambda}$ sin θ . (36-21)
centers of the slits and *a* is the slit width. Note
-19 is the product of I_m and two factors. (1) The
interference between two slits with slit separa-
-23). (2) The *diffraction factor* and $\alpha = \frac{1}{\lambda}$ sin θ . (36-21)

Here *d* is the distance between the centers of the slits and *a* is the slit width. Note

carefully that the right side of Eq. 36-19 is the product of I_m and two factors (1) The
 i 0 and $\cos^2 \beta = 1$. In this case Eq. 36-19 reduces, as it must, to an equation describing distance between the centers of the slits and *a* is the slit width. Note
the right side of Eq. 36-19 is the product of I_m and two factors. (1) The
ctor cos² β is due to the interference between two slits with sl the diffraction pattern for a single slit of width a. fully that the right side of Eq. 36-19 is the product of I_m and two factors. (1) The *ference factor* $\cos^2 \beta$ is due to the interference between two slits with slit separa-
d (as given by Eqs. 35-22 and 35-23). (2) T interference factor cos² β is due to the interference between two slits with slit separation d (as given by Eqs. 35-22 and 35-23). (2) The diffraction factor $[(\sin \alpha)/\alpha]^2$ is due to diffraction by a single slit of w tion *d* (as given by Eqs. 35-22 and 35-23). (2) The *diffraction factor* [(sin *a*)/ α]² is due to diffraction by a single slit of width *a* (as given by Eqs. 36-5 and 36-6). Let us check these factors. If we let $a \$ to diffraction by a single slit of width a (as given by Eqs. 36-5 and 36-6).

Let us check these factors. If we let $a \rightarrow 0$ in Eq. 36-21, for example, then $\alpha \rightarrow 0$ and (sin α)/ $\alpha \rightarrow 1$. Equation 36-19 then reduces, as Let us check these factors. If we let $a \rightarrow 0$ in Eq. 36-21, for example, then $\alpha \rightarrow 0$ and $(\sin \alpha)/\alpha \rightarrow 1$. Equation 36-19 then reduces, as it must, to an equation disearching the inteference pattern for a pair of vanishing

process interference. If the combining waves originate in a single wavefront—as in a single-slit experiment—we call the process diffraction. This distinction between interference and diffraction (which is somewhat arbitrary and not always adhered 36-4 DIFFRACTION BY A DOU
process *interference*. If the combining waves originate in a single wavefront—as in
a single-slit experiment—we call the process *diffraction*. This distinction between
interference and diffracti 36-4 DIFFRACTION BY A DO
process *interference*. If the combining waves originate in a single wavefront—as in
a single-slit experiment—we call the process *diffraction*. This distinction between
interference and diffractio process *interference*. If the combining waves originate in a single wavefron
a single-slit experiment—we call the process *diffraction*. This distinction
interference and diffraction (which is somewhat arbitrary and not process *interference*. If the combining waves originate in a single wavefront—as in
a single-slit experiment—we call the process *diffraction*. This distinction between
interference and diffraction (which is somewhat arb process *interference*. It the comoning waves originate in a single waventum — as
a single-slit experiment — we call the process *diffraction*. This distinction betwee
interference and diffraction (which is somewhat arbit

Sample Problem 36.05 Double-slit experiment with diffraction of each slit included

light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope? central peak of the diffraction envelope?

KEY IDEAS

We first analyze the two basic mechanisms responsible for the optical pattern produced in the experiment:

1. Single-slit diffraction: The limits of the central peak are the first minima in the diffraction pattern due to either slit individually. (See Fig. 36-15.) The angular locations of How many bright interference fringes are within the

thral peak of the diffraction envelope?
 $\begin{array}{c}\n\vdots \text{First analyze the two basic mechanisms responsible for}\n\end{array}$
 $\begin{array}{c}\n\vdots \text{first analyze the two basic mechanisms responsible for}\n\end{array}$
 $\begin{array}{c}\n\vdots \text{first analyze the two basic mechanisms responsible for}\n\end{array}$
 $\begin{array}{c}\n\vdots \text{first analyze the two basic mechanisms responsible for}\n\end{$ let us rewrite this equation as a sin $\theta = m_1 \lambda$, with the **EY IDEAS**

First analyze the two basic mechanisms responsible for

optical pattern produced in the experiment:

Single-slit diffraction: The limits of the central peak are

the first minima in the diffraction pattern due **EY IDEAS**

First analyze the two basic mechanisms responsible for

coptical pattern produced in the experiment:

Single-slit diffraction: The limits of the central peak are

the first minima in the diffraction pattern du minima in the diffraction pattern, we substitute $m_1 = 1$, fringe are shown in Fig. 36-17. obtaining mechanisms responsible for

the experiment:

in the experiment:

in the central peak are

in pattern due to either slit

.) The angular locations of

.) The angular locations of

the first and second side peaks of

the fi the first minima in the diffraction pattern due to either slit
individually. (See Fig. 36-15.) The angular locations of
those minima are given by Eq. 36-3 (*a* sin $\theta = m\lambda$). Here
let us rewrite this equation as *a* sin See Fig. 36-15.) The angular locations of
are given by Eq. 36-3 (*a* sin $\theta = m\lambda$). Here
this equation as *a* sin $\theta = m_1\lambda$, with the
erring to the one-slit diffraction. For the first
exitence: the instead of the double-The angular locations of
 $6-3$ ($a \sin \theta = m\lambda$). Here
 $a \sin \theta = m\lambda$, with the

it diffraction. For the first

m, we substitute $m_1 = 1$, fringe are shown in Fig. 36-17

(36-22) (b) How many bright fringes

(36-22) (b) How ma

$$
a\sin\theta = \lambda.\tag{36-22}
$$

2. Double-slit interference: The angular locations of the bright fringes of the double-slit interference pattern are

$$
d \sin \theta = m_2 \lambda, \quad \text{for } m_2 = 0, 1, 2, \dots. \quad (36-23)
$$

Here the subscript 2 refers to the double-slit interference.

Calculations: We can locate the first diffraction minimum obtaining

a sin $\theta = \lambda$. (36-22) (b) How many bright fring

2. Double-slit interference: The angular locations of the

bright fringes of the double-slit interference pattern are

given by Eq. 35-14, which we can write as a sin $\theta = \lambda$.

2. *Double-slit interference:* The angular local bright fringes of the double-slit interference given by Eq. 35-14, which we can write as $d \sin \theta = m_2 \lambda$, for $m_2 = 0, 1, 2, ...$

Here the subscript 2 refers to 9 = λ. (36-22) (b) F

side p

le-slit interference pattern are

we can write as

we can write as

x r m₂ = 0,1,2,.... (36-23) The

secor

rs to the double-slit interference. given

e the first diffraction minimum

patt 2. *Double-slit interference:* The angular locations of the bright fringes of the double-slit interference pattern are given by Eq. 35-14, which we can write as $d \sin \theta = m_2 \lambda$, for $m_2 = 0, 1, 2, \dots$ (36-23) Here the subscr

$$
m_2 = \frac{d}{a} = \frac{19.44 \,\mu\text{m}}{4.050 \,\mu\text{m}} = 4.8.
$$

This tells us that the bright interference fringe for $m_2 = 4$ fits into the central peak of the one-slit diffraction pattern, but the fringe for $m_2 = 5$ does not fit. Within the central dif-Frefers to the double-slit interference. given by a sin

locate the first diffraction minimum

inge pattern by dividing Eq. 36-23 by

or m_2 . By doing so and then substitut-

before the bright

interference fringe for fraction peak we have the central bright fringe $(m_2 = 0)$, Fig. 36-17). However, if the $m_2 = 5$ bright fringe, which is al-
and four bright fringes (up to $m_2 = 4$) on each side of it. most eliminated by the first diffra and four bright fringes (up to $m_2 = 4$) on each side of it. most within the double-slit fringe pattern by dividing Eq. 36-23 by
Eq. 36-22 and solving for m_2 . By doing so and then substitut-
ing the given data, we obtain
 $m_2 = \frac{d}{a} = \frac{19.44 \ \mu \text{m}}{4.050 \ \mu \text{m}} = 4.8$.
This tells u ence pattern are within the central peak of the diffraction

Figure 36-17 One side of the intensity plot for a two-slit interference experiment.The inset shows (vertically expanded) the plot within

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

KEY IDEA

The outer limits of the first side diffraction peaks are the the first and second side peaks of the diffraction envelope.

envelope. The bright fringes to one side of the central bright

fringe are shown in Fig. 36-17.

(b) How many bright fringes are within either of the first

si given by $a \sin \theta = m_1 \lambda$ with $m_1 = 2$: 2: one side of the central bright
are within either of the first
welope?
de diffraction peaks are the
the of which is at the angle θ
= 2:
2 λ . (36-24)
3 by Eq. 36-24, we find
 (9.44 cm) Chinese and the same of the first

side peaks of the diffraction envelope?
 KEY IDEA

The outer limits of the first side diffraction peaks are the

second diffraction minima, each of which is at the angle θ

given by

$$
a\sin\theta = 2\lambda.\tag{36-24}
$$

$$
m_2 = \frac{2d}{a} = \frac{(2)(19.44 \text{ }\mu\text{m})}{4.050 \text{ }\mu\text{m}} = 9.6.
$$

This tells us that the second diffraction minimum occurs just 4.8. before the bright interference fringe for $m_2 = 10$ in Eq. 36-23. peaks are the

at the angle θ

(36-24)

., we find

.6.

num occurs just

10 in Eq. 36-23.

ave the fringes

right fringes of

in the inset of Within either first side diffraction peak we have the fringes 4 from $m_2 = 5$ to $m_2 = 9$, for a total of five mma, each of which is at the angle θ
with $m_1 = 2$:
 $a \sin \theta = 2\lambda$. (36-24)
 $d\xi = 2$, (36-23 by Eq. 36-24, we find
 $= \frac{(2)(19.44 \mu m)}{4.050 \mu m} = 9.6$.
econd diffraction minimum occurs just
ference fringe for $m_2 = 10$ in E the double-slit interference pattern (shown in the inset of *a* sin $\theta = 2\lambda$. (36-24)
 Calculation: Dividing Eq. 36-23 by Eq. 36-24, we find
 $m_2 = \frac{2d}{a} = \frac{(2)(19.44 \,\mu\text{m})}{4.050 \,\mu\text{m}} = 9.6$.

This tells us that the second diffraction minimum occurs just

before the bright 5 bright fringes are in $(36-24)$

5 bright friend $\frac{14 \mu m}{\mu m}$ = 9.6.

5 continum in the space of $m_2 = 10$ in Eq. 36-23.

5 in peak we have the fringes

5 al of five bright fringes of

5 bright fringe, which is al-
 Calculation: Dividing Eq. 36-23 by Eq. 36-24, we find
 $m_2 = \frac{2d}{a} = \frac{(2)(19.44 \mu \text{m})}{4.050 \mu \text{m}} = 9.6.$

This tells us that the second diffraction minimum occurs just

before the bright interference fringe for $m_2 =$ **Calculation:** Dividing Eq. 36-23 by Eq. 36-24, we find
 $m_2 = \frac{2d}{a} = \frac{(2)(19.44 \mu \text{m})}{4.050 \mu \text{m}} = 9.6.$

This tells us that the second diffraction minimum occurs just

before the bright interference fringe for $m_2 =$ the first side diffraction peak. Fig. 36-17). However, if the $m_2 = 5$ bright fringe, which is al-

PLUS Additional examples, video, and practice available at WileyPLUS

36-5 DIFFRACTION GRATINGS

Learning Objectives

After reading this module, you should be able to . . .

- **36.22** Describe a diffraction grating and sketch the interference pattern it produces in monochromatic light.
- 36.23 Distinguish the interference patterns of a diffraction grating and a double-slit arrangement.
- 36.24 Identify the terms line and order number.
- 36.25 For a diffraction grating, relate order number m to the path length difference of rays that give a bright fringe.
- **36.26** For a diffraction grating, relate the slit separation d , the angle θ to a bright fringe in the pattern, the order number

Key Idea

● A diffraction grating is a series of "slits" used to separate an incident wave into its component wavelengths by separating and displaying their diffraction maxima. Diffraction by N (multiple) slits results in maxima (lines) at angles θ such that gth difference of rays that give a bright
tion grating, relate the slit separation d, the
ight fringe in the pattern, the order number
ating is a series of "slits" used to separate
into its component wavelengths by separa belate of the Hillsel *M*

of rays that give a bright

elate the slit separation *d*, the

elate the slit separation *d*, the

in a diffraction-gra

a diffraction-gra

a diffraction-gra

of "slits" used to separate

of "s

 $d \sin \theta = m\lambda$, for $m = 0, 1, 2, \dots$ (maxima).

 m of that fringe, and the wavelength λ of the light.

- **36.27** Identify the reason why there is a maximum order number for a given diffraction grating.
- 36.28 Explain the derivation of the equation for a line's half-width in a diffraction-grating pattern.
- 36.29 Calculate the half-width of a line at a given angle in a diffraction-grating pattern.
- 36.30 Explain the advantage of increasing the number of slits in a diffraction grating.
- 36.31 Explain how a grating spectroscope works.

● A line's half-width is the angle from its center to the point where it disappears into the darkness and is given by

$$
\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}
$$
 (half-width).

screen C.

$_P$ Diffraction Gratings

One of the most useful tools in the study of light and of objects that emit and absorb light is the diffraction grating. This device is somewhat like the double-slit **arrangement of Fig. 35-10 but has a much greater number of slits.** (maxima).
 Diffraction by N
 **One of the most useful tools in the study of light and of objects that emit and

absorb light is the diffraction grating Diffraction Gratings**

(maxima).

(maxima).

(maxima).
 Diffraction Gratings

One of the most useful tools in the study of light and of objects that emit and

absorb light is the **diffraction grating**. This device is s **Considered in the Consisting of only five slits is represented in Fig. 36-18.** When monochromation of the most useful tools in the study of light and of objects that emit and absorb light is the **diffraction grating**. Th **Inffraction Gratings**

One of the most useful tools in the study of light and of objects that emit and

absorb light is the **diffraction grating**. This device is somewhat like the double-slit

arrangement of Fig. 35-10 b **Diffraction Gratings**

One of the most useful tools in the study of light and of objects that emit and

absorb light is the **diffraction grating**. This device is somewhat like the double-slit

arrangement of Fig. 35-10 bu be opaque surfaces with narrow parallel grooves arranged like the slits in **Diffraction Gratings**

One of the most useful tools in the study of light and of objects that emit and

absorb light is the **diffraction grating**. This device is somewhat like the double-slit

arrangement of Fig. 35-10 b rather than being transmitted through open slits.) of the most useful tools in the study of light and of objects that emit and
orb light is the **diffraction grating**. This device is somewhat like the double-slit
negement of Fig. 35-10 but has a much greater number N of sl One of the most useful tools in the study of light and of objects that emit and
absorb light is the **diffraction grating**. This device is somewhat like the double-slit
arrangement of Fig. 35-10 but has a much greater numb absorb light is the **diffraction grating**. This device is somewhat like the double-slit arrangement of Fig. 35-10 but has a much greater number N of slits, often called *rulings*, perhaps as many as several thousand per m arrangement of Fig. 35-10 but has a much greater number N of slits, often called *rulings*, perhaps as many as several thousand per millimeter. An idealized grating consisting of only five slits is represented in Fig. 36-1 \ddot{c} Fig. 36-18. Light then scatters back from the grooves to form interference fringes

pattern you would see on a viewing screen using monochromatic red light from, Figure 36-18 An idealized diffraction grating,
consisting of only five rulings that produces *Pattern*. With monochromatic light incident on a diffraction grating, if we an interference pattern on a distant viewing gradually increase the number of slits from two to a large number N , the intensity plot changes from the typical double-slit plot of Fig. 36-15 c to a much more compli-

> **Figure 36-19** (a) The intensity plot produced by a diffraction grating with a great many corresponding bright fringes seen on the screen are called lines and are here also **Figure 36-19** (*a*) The intensity plot produced by a diffraction grating with a great many rulings consists of narrow peaks, here labeled with their order numbers *m*. (*b*) The corresponding bright fringes seen on the s

say, a helium–neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow
(and so are called *lines*); they are separated by relatively wide dark regions.
Equation. We use a familiar procedure to find the locat

36-5
say, a helium–neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow
(and so are called *lines*); they are separated by relatively wide dark regions.
Equation. We use a familiar procedure to find the l Equation. We use a familiar procedure to find the locations of the bright lines 36-

Say, a helium – neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow

(and so are called *lines*); they are separated by relatively wide dark regions.
 Equation. We use a familiar procedure to find t ing so that the rays reaching a particular point P on the screen are approximately say, a helium – neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow
(and so are called *lines*); they are separated by relatively wide dark regions.
Equation. We use a familiar procedure to find the loc adjacent rulings the same reasoning we used for double-slit interference.The separation d between rulings is called the grating spacing.(If N rulings occupy a total say, a helium–neon laser is shown in Fig. 36-19*b*. Th
(and so are called *lines*); they are separated by relation
Equation. We use a familiar procedure to find
on the viewing screen. We first assume that the screen
ing width w, then $d = w/N$.) The path length difference between adjacent rays is say, a helium-neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow (and so are called *lines*); they are separated by relatively wide dark regions.
 Equation. We use a familiar procedure to find the loca say, a helium – neon laser is shown in Fig. 36-19*b*. The maxima are now very narrow

(and so are called *lines*); they are separated by relatively wide dark regions.
 Equation. We use a familiar procedure to find the l length difference between adjacent rays is an integer number of wavelengths: a familiar procedure to find the locations of the
We first assume that the screen is far enough free
ching a particular point *P* on the screen are ap
we the grating (Fig. 36-20). Then we apply to α
me reasoning we use Exerces in the locations of the bright lines

Exerces it at the screen is far enough from the grati-

lar point P on the screen are approximately

(Fig. 36-20). Then we apply to each pair of

ve used for double-slit inter ing so that the rays reaching a particular point *P* on the screen are approximately
parallel when they leave the grating (Fig. 36-20). Then we apply to each pair of
adjacent rulings the same reasoning we used for doubleparallel when they leave the grating (Fig. 36-20). Then we apply to each pair of
adjacent rulings the same reasoning we used for double-slit interference. The sep-
width w, then $d = w/N$.) The path length difference between aration *d* between rulings is called the *grating spacing*. (If *l* width *w*, then $d = w/N$.) The path length difference be again *d* sin θ (Fig. 36-20), where θ is the angle from the central (and of the diffraction called the *grating spacing*. (If *N* rulings occupy a total
the path length difference between adjacent rays is
here θ is the angle from the central axis of the grating
ern) to point *P*. A line will be located at *P* f N rulings occupy a total

between adjacent rays is

entral axis of the grating

e located at P if the path

mber of wavelengths:

-lines), (36-25)

Figure 36-20 The r

presents a different line; diffraction gratin

pr

 $d \sin \theta = m\lambda$, for $m = 0, 1, 2, \dots$ (maxima—lines),

are then called the *order numbers*, and the lines are called the zeroth-order line ference between each two adjacent rays is
(the control line with $m = 0$) the first order line $(m = 1)$ the second order line ds in θ , w $(m = 2)$, and so on. *w*, then $d = w/N$.) The path length differ *d* sin θ (Fig. 36-20), where θ is the angle from of the diffraction pattern) to point *P*. A line difference between adjacent rays is an integ *d* sin $\theta = m\lambda$, for $m = 0,$ n *d* sin θ (Fig. 36-20), where θ is the angle from the central axis of the grating

1 of the diffraction pattern) to point *P*. A line will be located at *P* if the path

th difference between adjacent rays is an i (and of the diffraction pattern) to point *P*. A line will be located at *P* if the path
length difference between adjacent rays is an integer number of wavelengths:

 $d \sin \theta = m\lambda$, for $m = 0, 1, 2, ...$ (maxima—lines),

where

Determining Wavelength. If we rewrite Eq. 36-25 as $\theta = \sin^{-1}(m\lambda/d)$, we length difference between adjacent rays is an integer number of wavelengths:
 $d \sin \theta = m\lambda$, for $m = 0, 1, 2, ...$ (maxima—lines), (36-25)

where λ is the wavelength of the light. Each integer m represents a different lin d sin $\theta = m\lambda$, for $m = 0, 1, 2, ...$ (maxima—lines), (36-25)

where λ is the wavelength of the light. Each integer m represents a different line; diffraction grating

hence these integers can be used to label the lines, **a** sin $\theta = m\lambda$, for $m = 0, 1, 2, ...$ (maxima-lines), (36-25) **F**

where λ is the wavelength of the light. Each integer *m* represents a different line; diffraction grating

hence these integers can be used to label the where λ is the wavelength of the light. Each integer *m* represents a different line;
hence these integers can be used to label the lines, as in Fig. 36-19. The integers
are then called the *order numbers*, and the lin where λ is the wavelength of the light. Each integer *m* represents a different line; diffraction grating hence these integers can be used to label the lines, as in Fig. 36-19. The integers approximately parts are call hence these integers can be used to label the lines, as in Fig. 36-19. The integers approximately para are then called the *order numbers*, and the lines are called the zeroth-order line ference between each (the central are then called the *order numbers*, and the lines are called the zeroth-order line

(the central line, with $m = 0$), the first-order line ($m = 1$), the second-order line

($m = 2$), and so on.
 Determining Wavelength. I fringes due to different wavelengths overlap too much to be distinguished. the three impless on the width of the lines. We define the wavelength of the lines.

assed Thus, when light of an unknown wavelength is sent through a diffraction

grating, measurements of the angles to the higher-order l detainting, when in the control of the angles to the higher-order lines can Eq. 36-25 to determine the wavelength. Even light of several unknetlengths can be distinguished and identified in this way. We cannot due double-

Width of the Lines

A grating's ability to resolve (separate) lines of different wavelengths depends on the which of the lines. We shall never different the expression for the *analy which* of the central line (the line for which $m = 0$) and then state an expression for the *I* versus θ like Fig. 36-19*a*. grammy, measurements of the algress to the imperi-order lines can be used in
Eq. 36-25 to determine the wavelength. Even light of several unknown wave-
lengths can be distinguished and identified in this way. We cannot do as being the angle $\Delta \theta_{hw}$ from the center of the line at $\theta = 0$ outward to where the double-slit arrangement of Module 35-2, even though the same equation
the double-slit arrangement of Module 35-2, even though the same equation
and wavelength dependence apply there. In double-slit interference, the b The double-sin at angeled of Module 35-2, even though the same equation
and wavelength dependence apply there. In double-slit interference, the bright
fringes due to different wavelengths overlap too much to be distinguis and wavelength dependence apply there. In doubte-sint interterence, the original
 Contains the central of the central of the central of the central line is measured from the central line (the lines We shall here of the widths are usually compared via half-widths.) **Propertion C A** partial in the consistent of different wavelengths depends on \mathbf{r} and then is measured from the central line (the lines. We shall here derive an expression for the *half-width* of line to the adj A grating's ability to resolve (separate) lines of different wavelengths depends on
the width of the lines. We shall here derive an expression for the *half-width* of
the central line to the central line (the line of whic the width of the lines. We shall here derive an expression for the *half-width* of
the central line (the line for which $m = 0$) and then state an expression for the N -versus θ like Fig. 3
as being the algher-order li

In Module 36-1 we were also concerned with the cancellation of a great many the central line (the line for which $m = 0$) and then state an expression for the
half-widths of the higher-order lines. We define the **half-width** of the central line
the line effectively ends and darkness effectively be half-widths of the higher-order lines. We define the **half-width** of the central line
the line are $\theta = 0$ outward to where
the line effectively ends and darkness effectively begins with the first minimum
(Fig. 36-21). At as being the angle $\Delta\theta_{hw}$ from the center of the line at $\theta = 0$ outward to where

line effectively ends and darkness effectively begins with the first minimum

(Fig. 36-21). At such a minimum, the N rays from the N sl and so the path length difference between the top and bottom rays here is (Fig. 36-21). At such a minimum, the N rays from the N slits of the grone another. (The actual width of the central line is, of course, $2(\Delta \ell)$ widths are usually compared via half-widths.)
In Module 36-1 we were also co Intensity, of educid Calculation of a great many

slit. We obtained Eq. 36-3, which,

um occurs where the path length

quals λ. For single-slit diffraction,

lost um occurs where the path length

quals λ. For single-slit In Module 36-1 we were also concerned with the cancellatio
rays, there due to diffraction through a single slit. We obtained
because of the similarity of the two situations, we can use
minimum here. It tells us that the f oncerned with the cancellation of a great many

ugh a single slit. We obtained Eq. 36-3, which,

two situations, we can use to find the first

first minimum occurs where the path length

tom rays equals λ . For single-s rays, there due to diffraction through a single slit. We obtained
because of the similarity of the two situations, we can use t
minimum here. It tells us that the first minimum occurs where
difference between the top and

$$
Nd\sin\Delta\theta_{\text{hw}} = \lambda. \tag{36-26}
$$

$$
\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd}
$$
 (half-width of central line). (36-27) (The angle in the right-hand side)

the grating

if the path

engths:
 $(36-25)$

Figure 36-20 The rays from the rulings

ifferent line;

diffraction grating to a distant point *F*

he integers

approximately parallel. The path leng

encoder line
 $d \sin \theta$, Figure 36-20 The rays from the rulings in a diffraction grating to a distant point P are approximately parallel.The path length difference between each two adjacent rays is rulings extend into and out of the page.)

Figure 36-21 The half-width $\Delta \theta_{\text{hw}}$ of the central line is measured from the center of that line to the adjacent minimum on a plot of

Figure 36-22 The top and bottom rulings of separated by Nd. The top and bottom rays passing through these rulings have a path $\overrightarrow{A\theta}_{\text{hw}}$

Mathematic Bottom ray

Pathength

Figure 36-22 The top and bottom rulings of

a diffraction grating of N rulings are

separated by Nd. The top and bottom rays

passing through these rulings have a path

l $\Delta \theta_{\text{hw}}$ is the angle to the first minimum. (The angle is here greatly exaggerated Path length

Figure 36-22 The top and bottom rulings of

a diffraction grating of N rulings are

separated by Nd. The top and bottom rays

passing through these rulings have a path

length difference of Nd sin $\Delta \theta_{\text{hw}}$

Figure 36-23 A simple type of grating specof light emitted by source S.

We state without proof that the half-width of any other line depends on its location relative to the central axis and is

$$
\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta} \quad \text{(half-width of line at } \theta\text{)}.
$$
 (36-28)

We state without proof that the half-width of any other line depends on its location relative to the central axis and is
 $\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}$ (half-width of line at θ). (36-28)

Note that for light of a given wave We state without proof that the half-width of any other line depends on its location relative to the central axis and is
 $\Delta \theta_{hw} = \frac{\lambda}{Nd \cos \theta}$ (half-width of line at θ). (36-28)

Note that for light of a given wavelen We state without proof that the half-width of any other line depends on its location relative to the central axis and is
 $\Delta \theta_{hw} = \frac{\lambda}{Nd \cos \theta}$ (half-width of line at θ). (36-28)

Note that for light of a given wavelen distinguish between wavelengths because its diffraction lines are narrower and so produce less overlap. $\Delta \theta_{hw} = \frac{\lambda}{Nd \cos \theta}$ (half-width of line at θ). (36-28)

Note that for light of a given wavelength λ and a given ruling separation *d*, the

widths of the lines decrease with an increase in the number *N* of ruling

Grating Spectroscope

Diffraction gratings are widely used to determine the wavelengths that are emit-Nd cos θ

Note that for light of a given wavelength λ and a given ruling separation d, the

widths of the lines decrease with an increase in the number N of rulings. Thus, of

two diffraction gratings, the grating w source S is focused by lens L_1 on a vertical slit S_1 placed in the focal plane of lens L_2 . The light emerging from tube C (called a *collimator*) is a plane wave and is incident perpendicularly on grating G, where it is diffracted into a diffraction be that ion fight or a given wavelength *A* and a given funding separation *a*, the dish of the lines decrease with an increase in the number *N* of rulings. Thus, of o diffraction gratings, the grating with the larger va when so the must decrease what an increase in the humber *N* of rumigs. Finst, or
two diffraction gratings, the grating with the larger value of *N* is better able to
distinguish between wavelengths because its diffractio two diffraction gratings, the grating with the large
distinguish between wavelengths because its diffrace
produce less overlap.
Grating Spectroscope
Diffraction gratings are widely used to determine t
ted by sources of 0 order diffracted at angle $\theta = 0$ along the central axis of the grating. **Grating Spectroscope**

Diffraction gratings are widely used to determine the wavelengths that are emit-

ted by sources of light ranging from lamps to stars. Figure 36-23 shows a simple

grating spectroscope in which a g Diffraction gratings are widely used to determine the wavelengths that are emit-
ted by sources of light ranging from lamps to stars. Figure 36-23 shows a simple
grating spectroscope in which a grating is used for this pu ted by sources of light ranging from lamps to stars. Figure 36-23 shows a si
grating spectroscope in which a grating is used for this purpose. Light
source S is focused by lens L_1 on a vertical slit S_1 placed in the ing spectroscope in which a grating is used for this purpose. Light from
ce *S* is focused by lens L_1 on a vertical slit S_1 placed in the focal plane of lens
The light emerging from tube *C* (called a *collimator*)

We can view the diffraction pattern that would appear on a viewing screen at the telescope then focuses the light diffracted at angle θ (and at slightly smaller source *S* is focused by lens L_1 on a vertical slit S_1 placed in the focal plane L_2 . The light emerging from tube *C* (called a *collimator*) is a plane wave incident perpendicularly on grating *G*, where it is d S_1 placed in the focal plane of lens

collimator) is a plane wave and is

e it is diffracted into a diffraction

gle $\theta = 0$ along the central axis of

ould appear on a viewing screen at

Fig. 36-23 to that angle. Lens

 L_2 . The light emerging from tube C (called a *collimator*) is a plane wave and is incident perpendicularly on grating G, where it is diffracted into a diffraction pattern, with the $m = 0$ order diffracted at angle $\theta =$ incident perpendicularly on grating *G*, where it is diffracted into a diffraction
pattern, with the $m = 0$ order diffracted at angle $\theta = 0$ along the central axis of
the grating.
We can view the diffraction pattern that pattern, with the $m = 0$ order diffracted at angle $\theta = 0$ along the central axis of
the grating.
If the cavacave we can view the diffraction pattern that would appear on a viewing screen at
any angle θ simply by orien the grating.
We can view the diffraction pattern that would appear on a viewing screen at
any angle θ simply by orienting telescope T in Fig. 36-23 to that angle. Lens L_3 of
the telescope then focuses the light diff shorter-wavelength line at a smaller angle θ than the longer-wavelength line. angle θ simply by orienting telescope T in Fig. 36-23 to that angle. Lens L_3 of telescope then focuses the light diffracted at angle θ (and at slightly smaller larger angles) onto a focal plane FF' within the te the telescope then focuses the light diffracted at angle θ (and at slightly smaller and larger angles) onto a focal plane FF' within the telescope. When we look through eyepiece E , we see a magnified view of this fo and larger angles) onto a focal plane FF' within the telescope. When we look
through eyepiece E, we see a magnified view of this focused image.
By changing the angle θ of the telescope, we can examine the entire diffr through eyepiece *E*, we see a magnified view of this focused image.
By changing the angle θ of the telescope, we can examine the entire diffraction
pattern. For any order number other than $m = 0$, the original light i transverse is simply style of analyze the wavelengths and the set of light emitted by source s.

the wavelengths of light emitted by source s.

By changing the angle θ of the telescope, we can examine the entire diffraction
pattern. For any order number other than $m = 0$, the original light is spread out ac-
cording to wavelength (or color) so that we can dete pattern. For any order number other than $m = 0$, the original light is spread out according to wavelength (or color) so that we can determine, with Eq. 36-25, just what leavelengths are being emitted by the source. If the out ac-
it what
wave-
angles
ith the
h con-
r eyes
ough a
e four
ission
0), the
single
trually cording to wavelength (or color) so that we can determine, with Eq. 36-25, just what wavelengths, what we see as we rotate the telescope horizontally through the angles leonfresponding to an order *m* is a vertical line o white line at $\theta = 0$. The colors are separated in the higher orders. ths, what we see as we rotate the telescope horizontally through the angles esponding to an order *m* is a vertical line of color for each wavelength, with the ter-wavelength line at a smaller angle θ than the longer-w corresponding to an order *m* is a vertical line of color for each wavelength, with the shorter-wavelength line at a smaller angle θ than the longer-wavelength line. **Hydrogen** I for example, the light emitted by a hyd shorter-wavelength line at a smaller angle θ than the longer-wavelength line.
 Hydrogen. For example, the light emitted by a hydrogen lamp, which con-

tains hydrogen gas, has four discrete wavelengths in the visible

36-6 GRATINGS: DISPERSION AND RESOLVING POWER 1101

Figure 36-25 The visible emission lines of cadmium, as seen through a grating spectroscope.

Department of Physics, Imperial College/Science Photo Library/ Photo Researchers, Inc.

36-6 **GRATINGS: DISPERSION AND RES**
 Solution Eq. 36-25 The visible

emission lines of cadmium,

as seen through a grating

pepartment of Physics, Imperial College/Science Photo Library/

spectroscope.

Photo Researcher **Since 36-25** The visible
sin in ines of cadmium,
as seen through a grating $\frac{1}{2}$
as seen through a grating $\frac{1}{2}$
solve Eq. 36-25 for the angle θ for the red wavelength when $m = 4$, we find that
sin θ is gre Figure 36-25 The visible
emission lines of cadmium,
as seen through a grating
pectroscope. Photo Researchers, Inc.
solve Eq. 36-25 for the angle θ for the red wavelength when $m = 4$, we find that
sin θ is greater th **Figure 36-25** The visible
emission lines of cadmium,
as seen through a grating
pectroscope. Photo Researchers, Inc.
solve Eq. 36-25 for the angle θ for the red wavelength when $m = 4$, we find that
sin θ is greater photograph of the visible emission lines produced by cadmium. Ive Eq. 36-25 for the angle θ for the red wavelength when $m = 4$, we find that θ is greater than unity, which is not possible. The fourth order is then said to be *complete* for this grating; it might not be incompl Ive Eq. 36-25 for the angle θ for the red wavelength when $m = 4$, we find it θ is greater than unity, which is not possible. The fourth order is then said to *complete* for this grating; it might not be incomplete f derstate than unity, which is not possible. The fourth order is then said to be *complete* for this grating; it might not be incomplete for a grating with greater acing *d*, which will spread the lines less than in Fig. 3

Checkpoint 5

The figure shows lines of different orders produced by the center of the pattern to the left or right? (b) In

36-6 GRATINGS: DISPERSION AND RESOLVING POWER

Learning Objectives

After reading this module, you should be able to . . .

- 36.32 Identify dispersion as the spreading apart of the diffraction lines associated with different wavelengths.
- **36.33** Apply the relationships between dispersion D , wavelength difference $\Delta \lambda$, angular separation $\Delta \theta$, slit separation d , order number m , and the angle θ corresponding to the order number. **Solution** differation interesting and the different the different interesting the angular separation *D*, wavelengths difference $\Delta \lambda$, angular separation *D*, **36.36** Apply wavelength difference $\Delta \lambda$, angular separat
- 36.34 Identify the effect on the dispersion of a diffraction

Key Ideas

 \bullet The dispersion D of a diffraction grating is a measure of wavelengths differing by $\Delta \lambda$. For order number m, at angle θ , the dispersion is given by

$$
D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta}
$$
 (dispersion).

grating if the slit separation is varied.

- 36.35 Identify that for us to resolve lines, a diffraction grating must make them distinguishable.
- 36.36 Apply the relationship between resolving power R , wavelength difference $\Delta \lambda$, average wavelength λ_{avg} , number of rulings N , and order number m .
- **36.37** Identify the effect on the resolving power R if the number of slits N is increased.

 \bullet The resolving power R of a diffraction grating is a measure of its ability to make the emission lines of two close wavelengths distinguishable. For two wavelengths differing by $\Delta\lambda$ and with an average value of λ_{avg} , the resolving power is given by

$$
R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm \quad \text{(resolving power)}.
$$

Gratings: Dispersion and Resolving Power

Dispersion

To be useful in distinguishing wavelengths that are close to each other (as in a We
dispersion is given by
the dispersion is given by
 $D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$ (dispersion).
 $R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm$ (resolving pc
 Gratings: Dispersion and Resolving Power

Dispersion

To be useful in distinguishing wa an average value of A_{avg} the
 $D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$ (dispersion).
 $R = \frac{\lambda_{\text{avg}}}{\Delta \lambda}$
 Gratings: Dispersion and Resolving Power

Dispersion

To be useful in distinguishing wavelengths that are close to eac

$$
D = \frac{\Delta \theta}{\Delta \lambda}
$$
 (dispersion defined). (36-29)

compact disc function as a diffraction gratdiffraction patterns from the rulings.

Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$. Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$.
The greater D is, the greater is the distance between two emission lines whose
wavelengths differ by $\Delta\lambda$. We show below that the di wavelengths differ by $\Delta\lambda$. We show below that the dispersion of a grating at angle θ is given by Here $\Delta \theta$ is the angular separation of two lines whose wavelengths differ by $\Delta \lambda$.
The greater *D* is, the greater is the distance between two emission lines whose
wavelengths differ by $\Delta \lambda$. We show below that the Here $\Delta\theta$ is the angular separation of two lines whose wavelengths differ by $\Delta\lambda$.
The greater *D* is, the greater is the distance between two emission lines whose
wavelengths differ by $\Delta\lambda$. We show below that the Here $\Delta \theta$ is the angular separation of two lines whose wavelengths differ by $\Delta \lambda$.
The greater *D* is, the greater is the distance between two emission lines whose
wavelengths differ by $\Delta \lambda$. We show below that the

$$
D = \frac{m}{d \cos \theta}
$$
 (dispersion of a grating). (36-30)

or the radian per meter. $B = \frac{m}{d \cos \theta}$ (dispersion of a grating). (36-30)

Thus, to achieve higher dispersion we must use a grating of smaller grating spac-

ing d and work in a higher-order m. Note that the dispersion does not depend on

the n $D = \frac{m}{d \cos \theta}$ (dispersion of a grating). (36-30)

Thus, to achieve higher dispersion we must use a grating of smaller grating spac-

ing d and work in a higher-order m. Note that the dispersion does not depend on

the n $D = \frac{m}{d \cos \theta}$ (dispersion of a grating). (36-30)
Thus, to achieve higher dispersion we must use a grating of smaller grating spac-
ing d and work in a higher-order m. Note that the dispersion does not depend on
the numb Kristen Brochmann/Fundamental Photographs the number of rulings N in the grating. The SI unit for D is the degree per meter

Resolving Power

$$
R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} \quad \text{(resolving power defined).} \tag{36-31}
$$

Here λ_{avg} is the mean wavelength of two emission lines that can barely be recognized as separate, and $\Delta \lambda$ is the wavelength difference between them. The greater **Resolving Power**

To *resolve* lines whose wavelengths are close together (that is, to make the lines

distinguishable), the line should also be as narrow as possible. Expressed other-

wise, the grating should have a hi **Resolving Power**

To *resolve* lines whose wavelengths are close together (that is, to make the lines

distinguishable), the line should also be as narrow as possible. Expressed other-

wise, the grating should have a hi below that the resolving power of a grating is given by the simple expression wise, the grating should have a high **resolving power** *R*, defined as
 $R = \frac{\lambda_{avg}}{\Delta \lambda}$ (resolving power defined). (36-3

Here λ_{avg} is the mean wavelength of two emission lines that can barely be reconized as separate Here λ_{avg} is the mean wavelength of two emission lines that can barely be recognized as separate, and $\Delta \lambda$ is the wavelength difference between them. The greater *R* is, the closer two emission lines can be and still

$$
R = Nm
$$
 (resolving power of a grating). (36-32)

Proof of Eq. 36-30

fraction pattern of a grating: below that the resolving power of a grating is given by the simple expression
 $R = Nm$ (resolving power of a grating). (36-32)

To achieve high resolving power, we must use many rulings (large N).
 Proof of Eq. 36-30

Le

$$
d\sin\theta = m\lambda.
$$

$$
d(\cos \theta) d\theta = m d\lambda.
$$

To achieve high resolving power, we must use many rulings (large N).
 Proof of Eq. 36-30

Let us start with Eq. 36-25, the expression for the locations of the lines in the dif-

fraction pattern of a grating:
 $d \sin \theta = m\$ obtaining

$$
d(\cos \theta) \Delta \theta = m \Delta \lambda \tag{36-33}
$$

.

Let us regard
$$
\theta
$$
 and λ as variables and take differentials of this equation. We
\n
$$
d(\cos \theta) d\theta = m d\lambda.
$$
\nFor small enough angles, we can write these differentials as small differed
\nobtaining
\n
$$
d(\cos \theta) \Delta \theta = m \Delta \lambda
$$
\n(36)
\nor
\n
$$
\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}.
$$
\nThe ratio on the left is simply *D* (see Eq. 36-29), and so we have indeed der
\nEq. 36-30.
\n**Proof of Eq. 36-32**
\nWe start with Eq. 36-32 which was derived from Eq. 36-25, the expression for

 $d \sin \theta = m\lambda$.

Let us regard θ and λ as variables and take differentials of this equation. We find
 $d(\cos \theta) d\theta = m d\lambda$.

For small enough angles, we can write these differentials as small differences,

obtaining
 $d(\cos \$

Proof of Eq. 36-32

For small enough angles, we can write these differentials as small differences,

obtaining
 $d(\cos \theta) \Delta \theta = m \Delta \lambda$ (36-33)

or
 $\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$.

The ratio on the left is simply *D* (see Eq. 36-29), and so we have obtaining $d(\cos \theta) \Delta \theta = m \Delta \lambda$ (36-33)

or $\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$.

The ratio on the left is simply D (see Eq. 36-29), and so we have indeed derived

Eq. 36-30.
 Proof of Eq. 36-32

We start with Eq. 36-33, which was small wavelength difference between two waves that are diffracted by the grator

or
 $\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$.

The ratio on the left is simply D (see Eq. 36-29), and so we have indeed derived

Eq. 36-30.
 Proof of Eq. 36-32

We start with Eq. 36-33, which was derived from Eq. 36-25, the expre or
 $\frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$.

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Eq. 36-30.
 Proof of Eq. 36-32

We start with Eq. 36-33, which was derived from Eq. 36-25, the express $\Delta \lambda = d \cos \theta$.

The ratio on the left is simply D (see Eq. 36-29), and so we have indeed derived

Eq. 36-30.
 Proof of Eq. 36-32

We start with Eq. 36-33, which was derived from Eq. 36-25, the expression for the

locatio The ratio on the left is simply D (see Eq. 36
Eq. 36-30.
Proof of Eq. 36-32
We start with Eq. 36-33, which was derived filocations of the lines in the diffraction patter
small wavelength difference between two w
ing, an

$$
\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}.
$$

						36-6 GRATINGS: DISPERSION AND RESOLVING POWI
	Table 36-1 Three Gratings ^a					
Grating	\boldsymbol{N}	d (nm)	θ	$D(^{\circ}/\mu m)$	$\mathbb R$	
\boldsymbol{A}	10000	2540	13.4°	23.2	10 000	
\boldsymbol{B}	20 000	2540	13.4°	23.2	20 000	
\mathcal{C}	10 000	1360	25.5°	46.3	10 000	
	"Data are for $\lambda = 589$ nm and $m = 1$.					
	If we substitute $\Delta \theta_{hw}$ as given here for $\Delta \theta$ in Eq. 36-33, we find that		Grating			
			$\frac{\lambda}{N} = m \Delta \lambda,$			Intensity А

If we substitute $\Delta \theta_{\text{hw}}$ as given here for $\Delta \theta$ in Eq. 36-33, we to $\frac{\lambda}{N} = m \Delta \lambda$,
from which it readily follows that $R = \frac{\lambda}{\Delta \lambda} = Nm$.
This is Eq. 36-32, which we set out to derive.
Dispersion and Resolving Po

$$
\frac{\lambda}{N} = m \Delta \lambda,
$$

from which it readily follows that

$$
R=\frac{\lambda}{\Delta\lambda}=Nm.
$$

Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused with its dispersion.
Table 36-1 shows the characteristics of three gratings, all illuminated with light of
wavelength, $y = 580$ pm whose diffracted light is viewed in $\frac{\lambda}{N} = m \Delta \lambda$,

from which it readily follows that
 $R = \frac{\lambda}{\Delta \lambda} = Nm$.

This is Eq. 36-32, which we set out to derive.
 Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused wi wavelength $\lambda = 589$ nm, whose diffracted light is viewed in the first order ($m = 1$) N N $\cdots \cdots N$,

Adily follows that
 $R = \frac{\lambda}{\Delta \lambda} = Nm$.

Which we set out to derive.
 EXECUTE:

Solving **Power Compared**

Solving **Power Compared**

Solving **Power Compared**

Solving **Power Compared**

Solving **Power Com** from which it readily follows that
 $R = \frac{\lambda}{\Delta \lambda} = Nm$.

This is Eq. 36-32, which we set out to derive.
 Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused with its dispersion. This is Eq. 36-32, which we set out to derive.

This is Eq. 36-32, which we set out to derive.
 $R = \frac{\lambda}{\Delta \lambda} = Nm$.

This is Eq. 36-32, which we set out to derive.
 $\frac{18}{5}$
 Dispersion and Resolving Power Compared

Th This is Eq. 36-32, which we set out to derive.
 Dispersion and Resolving Power Compared

The resolving power of a grating must not be confused with its dispersion.

Table 36-1 shows the characteristics of three gratings For the conditions noted in Table 36-1, gratings A and B have the same of the conducted by these eratings for two lines of waveleneths λ , and λ , in the same of the set eratings and the same the same of α . The sam

dispersion D and A and C have the same resolving power R .

Figure 36-26 shows the intensity patterns (also called *line shapes*) that would state is esselving power, and grating C the Figure 36-26 shows the intensity patterns (also called *line shapes*) that would be produced by these gratings for two lines of wavelengths λ_1 and λ_2 , in the vicinity of $\lambda = 589$ nm. Grating B, with the higher resolving power, produces **Example 100 The complete**

Sube characteristics of three gratings, all illuminated with light of

589 nm, whose diffracted light is viewed in the first order $(m = 1$

ou should verify that the values of *D* and *R* as giv narrower lines and thus is capable of distinguishing lines that are much closer Table 36-1 shows the characteristics of three gratings, all illuminated with light of
wavelength $\lambda = 589$ nm, whose diffracted light is viewed in the first order $(m = 1$
in Eq. 36-25). You should verify that the values of wavelength $\lambda = 589$ nm, whose diffracted light is viewed in the first order (*n*.
in Eq. 36-25). You should verify that the values of *D* and *R* as given in the can be calculated with Eqs. 36-30 and 36-32, respectively. be produced by these gratings for two lines of wavelengths λ_1 and λ_2 , in the vicinity of $\lambda = 589$ nm. Grating *B*, with the higher resolving power, produces narrower lines and thus is capable of distinguishing li 25.4 mm. It is illuminated at normal state
of distinguishing lines that are
length than those in the figure. Grating *C*, with the
sthe greater angular separation between the lines.
Dispersion and resolving povertion 36.

Figure 36-26 The intensity patterns for light of two wavelengths sent through the grathighest dispersion.

Sample Problem 36.06 Dispersion and resolving power of a diffraction grating
A diffraction grating has 1.26×10^4 rulings uniformly spaced
over width $w = 25.4$ mm. It is illuminated at normal inci-
dence by yellow li over width $w = 25.4$ mm. It is illuminated at normal incimarrower lines and thus is capable of distinguishing lines that are much closer
together in wavelength than those in the figure. Grating C, with the higher dis-
persion, produces the greater angular separation between the contains two closely spaced emission lines (known as the persion, produces the greater angular separation between the lines.
 Sample Problem 36.06 Dispersion and resolving power of a diffraction grated

A diffraction grating has 1.26×10^4 rulings uniformly spaced

over wi **Sample Problem 36.06 Dispersion and resolving**
A diffraction grating has 1.26×10^4 rulings uniformly spaced
over width $w = 25.4$ mm. It is illuminated at normal inci-
dence by yellow light from a sodium vapor lamp. T

(a) At what angle does the first-order maximum occur (on the differential of the center of the diffraction pattern) for the differential these values for d and m into Eq. 36-25 leads to either side of the center of the diffraction pattern) for the

KEY IDEA

The maxima produced by the diffraction grating can be determined with Eq. 36-25 (d sin $\theta = m\lambda$). **Calculations:** The grating spacing d is

$$
d = \frac{w}{N} = \frac{25.4 \times 10^{-3} \text{ m}}{1.26 \times 10^{4}}
$$

= 2.016 × 10⁻⁶ m = 2016 nm.

$$
= 2.016 \times 10^{-6} \,\mathrm{m} = 2016 \,\mathrm{nm}.
$$

Calculations: The grating spacing d is
 $d = \frac{w}{N} = \frac{25.4 \times 10^{-3} \text{ m}}{1.26 \times 10^{4}}$
 $= 2.016 \times 10^{-6} \text{ m} = 2016 \text{ nm}.$

The first-order maximum corresponds to $m = 1$. Substituting

these values for d and m into Eq. 36 **Calculations:** The grating spacing d is
 $d = \frac{w}{N} = \frac{25.4 \times 10^{-3} \text{ m}}{1.26 \times 10^4}$
 $= 2.016 \times 10^{-6} \text{ m} = 2016 \text{ nm}.$

The first-order maximum corresponds to $m = 1$. Substituting

these values for d and m into Eq. 36 The first-order maximum corresponds to $m = 1$. Substituting

Corcreationalities: The grating opening a is

\n
$$
d = \frac{w}{N} = \frac{25.4 \times 10^{-3} \text{ m}}{1.26 \times 10^{4}} = 2.016 \times 10^{-6} \text{ m} = 2016 \text{ nm}.
$$
\nThe first-order maximum corresponds to $m = 1$. Substituting these values for d and m into Eq. 36-25 leads to

\n
$$
\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(589.00 \text{ nm})}{2016 \text{ nm}} = 16.99^{\circ} \approx 17.0^{\circ}.
$$
\n(Answer)

\n(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

(b) Using the dispersion of the grating, calculate the angular separation between the two lines in the first order.

KEY IDEAS

(1) The angular separation $\Delta\theta$ between the two lines in the first order depends on their wavelength difference $\Delta \lambda$ and 1104 CHAPTER 36 DIFFRACTION

(c) What is the least number

(1) The angular separation $\Delta \theta$ between the two lines in the

first order depends on their wavelength difference $\Delta \lambda$ and

the dispersion D of the grating, ac $(D = \Delta \theta / \Delta \lambda)$. (2) The dispersion D depends on the angle θ **IDEAS**

u. CHAPTER 36 DIFFRACTION

u. (c) What is the

e angular separation $\Delta \theta$ between the two lines in the

order?

ispersion D of the grating, according to Eq. 36-29
 $\Delta \theta/\Delta \lambda$). (2) The dispersion D depends on th at which it is to be evaluated. **ITO A CHAPTER 36 DIFFRACTION**

(1) The angular separation $\Delta \theta$ between the two lines in the

first order depends on their wavelength difference $\Delta \lambda$ and

the dispersion D of the grating, according to Eq. 36-29

($D = \$ **EXEY IDEAS**

(1) The angular separation $\Delta \theta$ between the two lines in the

first order depends on their wavelength difference $\Delta \lambda$ and

the dispersion D of the grating, according to Eq. 36-29

($D = \Delta \theta / \Delta \lambda$). (2) Th **ICENTE AS**

(1) The angular separation $\Delta\theta$ between the two lines in the

first order depends on their wavelength difference $\Delta\lambda$ and

the dispersion *D* of the grating, according to Eq. 36-29

($D = \Delta\theta/\Delta\lambda$). (2) Th

Calculations: We can assume that, in the first order, the

at which it is to be evaluated.

The resolving po

two social methands of the social correlations: We can assume that, in the first order, the

evaluate D at the = 16.99° we found in part (a) for

1.36-30 gives the dispersion as
 $\frac{1}{(2016 \text{ nm})(\cos 16.99^\circ)}$

$$
D = \frac{m}{d \cos \theta} = \frac{1}{(2016 \text{ nm})(\cos 16.99^\circ)}
$$

= 5.187 × 10⁻⁴ rad/nm.

$$
\Delta \theta = D \Delta \lambda = (5.187 \times 10^{-4} \text{ rad/nm})(589.59 - 589.00)
$$

= 3.06 × 10⁻⁴ rad = 0.0175°. (Answer)

You can show that this result depends on the grating spacing $=$ $= 3.06 \times 10^{-4}$ rad $= 0.0175^{\circ}$. (Answer)
You can show that this result depends on the grating spacing *d* but not on the number of rulings there are in the grating.

(c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?

KEY IDEAS

two sodium lines occur close enough to each other for us to according to Eq. 36-32 $(R = Nm)$. (2) The smallest wave-(c) What is the least number of rulings a grating can have
and still be able to resolve the sodium doublet in the first
order?
KEY IDEAS
(1) The resolving power of a grating in any order *m* is physi-
cally set by the n cally set by the number of rulings N in the grating (c) What is the least number of rulings a grating can have and still be able to resolve the sodium doublet in the first order?
 KEY IDEAS

(1) The resolving power of a grating in any order *m* is physelly set by the num of rulings a grating can have

e sodium doublet in the first

rating in any order m is physi-

f rulings N in the grating

Nm). (2) The smallest wave-

be resolved depends on the

and on the resolving power R

36-31 ($R = \$ length difference $\Delta \lambda$ that can be resolved depends on the average wavelength involved and on the resolving power R (c) What is the least number of rulings a grating can have
and still be able to resolve the sodium doublet in the first
order?
KEY IDEAS
(1) The resolving power of a grating in any order *m* is physi-
cally set by the n of the grating, according to Eq. 36-31 ($R = \lambda_{\text{avg}}/\Delta\lambda$). **ICEY IDEAS**

(1) The resolving power of a grating in any order *m* is physically set by the number of rulings *N* in the grating according to Eq. 36-32 ($R = Nm$). (2) The smallest wavelength difference $\Delta \lambda$ that can be

 $= 5.187 \times 10^{-4}$ rad/nm.
From Eq. 36-29 and with $\Delta \lambda$ in nanometers, we then have find that the smallest number of rulings for a grating to resolve the sodium doublet is Calculation: For the sodium doublet to be barely resolved, find that the smallest number of rulings for a grating to resolve the sodium doublet is

$$
N = \frac{R}{m} = \frac{\lambda_{\text{avg}}}{m \Delta\lambda}
$$

= $\frac{589.30 \text{ nm}}{(1)(0.59 \text{ nm})} = 999 \text{ rulings.}$ (Answer)

PLUS Additional examples, video, and practice available at WileyPLUS

36-7 X-RAY DIFFRACTION

Learning Objectives

After reading this module, you should be able to ...

36.38 Identify approximately where x rays are located in the electromagnetic spectrum.

36.39 Define a unit cell.

- 36.40 Define reflecting planes (or crystal planes) and interplanar spacing.
- 36.41 Sketch two rays that scatter from adjacent planes, showing the angle that is used in calculations.

Key Ideas

● If x rays are directed toward a crystal structure, they undergo Bragg scattering, which is easiest to visualize if **Example 18 Solution**
 S6.39 Define a unit cell.
 S6.40 Define reflecting planes (or crystal planes) and

interplanar spacing.
 S6.41 Sketch two rays that scatter from adjacent planes,

showing the angle that is use

 \bullet For x rays of wavelength λ scattering from crystal planes

 $e \theta$ of scattering,
avelength λ of the
all, demonstrate how an
armined.
which the scattered
1,2,3,... (Bragg's law). **36.42** For the intensity maxima in x-ray scattering **42** For the intensity maxima in x-ray scattering
by a crystal, apply the relationship between the
interplanar spacing d , the angle θ of scattering,
the order number m , and the wavelength λ of the
x rays. interplanar spacing d , the angle θ of scattering, the order number m , and the wavelength λ of the x rays.

36.43 Given a drawing of a unit cell, demonstrate how an interplanar spacing can be determined.

with separation d , the angles θ at which the scattered intensity is maximum are given by

 $2d \sin \theta = m\lambda$. for $m = 1, 2, 3, \ldots$ (Bragg's law).

X-Ray Diffraction

ture, they with separation d, the angles θ at which the scattered

allel planes.

2d sin $\theta = m\lambda$, for $m = 1, 2, 3, ...$ (Bragg's law).
 X-Ray Diffraction

X rays are electromagnetic radiation whose wavelengths are of t $(= 10^{-10} \text{ m})$. Compare this with a wavelength of 550 nm $(= 5.5 \times 10^{-7} \text{ m})$ at the 1, they with separation *d*, the angles θ at which the scattered

1 planes.

1 planes.

2*d* sin $\theta = m\lambda$, for $m = 1, 2, 3, ...$ (Bragg's law).
 Ray Diffraction

1 and 22 nm (- 1, 2, 3, ... (Bragg's law).
 Ray Diffract

36-7 X-RAY DIFF
center of the visible spectrum. Figure 36-27 shows that x rays are produced when
electrons escaping from a heated filament F are accelerated by a potential differ-
ence V and strike a metal target T.
A stan electrons escaping from a heated filament F are accelerated by a potential difference V and strike a metal target T.

A standard optical diffraction grating cannot be used to discriminate between different wavelengths in the x-ray wavelength range. For $\lambda = 1$ A standard optical diffraction grating cannot be used to discriminate between different wavelengths in the x-ray wavelength range. For $\lambda = 1$ Å $(=$ $(= 0.1 \text{ nm})$ and $d = 3000 \text{ nm}$, for example, Eq. 36-25 shows that the first-order ter of the visible spectrum. Figure 36-27 show
trons escaping from a heated filament *F* are
e V and strike a metal target *T*.
A standard optical diffraction grating c
ween different wavelengths in the x-ray
0.1 nm) and 36-7 X-RAY

spectrum. Figure 36-27 shows that x rays are produced when

om a heated filament F are accelerated by a potential differ-

etal target T.

cial diffraction grating cannot be used to discriminate

avelengths in maximum occurs at center of the visible spectrum. Figure 36-27 shows that x rays are produced when
electrons escaping from a heated filament F are accelerated by a potential differ-
ence V and strike a metal target T.
A standard optical di electrons escaping from a heated filament *F* are accelerated by a potential difference *V* and strike a metal target *T*.

A standard optical diffraction grating cannot be used to discriminate

between different waveleng A standard ance and angular 1.

A standard optical diffraction grating cannot be used to discriminate

veen different wavelengths in the x-ray wavelength range. For $\lambda = 1 \text{ Å}$

1.1 nm) and $d = 3000 \text{ nm}$, for example, E

$$
\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} = 0.0019^{\circ}.
$$

such gratings cannot be constructed mechanically.

Example the dimension of grading calling to the set to discriminate

(= 0.1 nm) and $d = 3000$ nm, for example, Eq. 36-25 shows that the first-order

maximum occurs at
 $\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} = 0.00$ $\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} = 0.0019^\circ.$ Figure 36-27 X rays and the sit of documentation grating $\theta = \sin^{-1} \frac{m\lambda}{3000 \text{ nm}} = 0.0019^\circ.$ Figure 36-27 X rays and the sit of documentation grating to be pra (-0.1 mm) and $u = 5000$ mm, for example, Eq. 50-25 shows that the first-order
maximum occurs at
maximum cocurs at
This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is
desirable, but, becau throughout the array. Figure 36-28a represents a section through a crystally with $d \approx \lambda$ is electrons desirable, but, because x-ray wavelengths are about equal to atomic diameters, accelerated performs a crystalline in 1 $\theta = \sin^{-1} \frac{m\lambda}{d} = \sin^{-1} \frac{(1)(0.1 \text{ nm})}{3000 \text{ nm}} = 0.0019^{\circ}.$

This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is

desirable, but, because x-ray wavelengths are about equal to atomic diamet $\theta = \sin^{-1} \frac{d\theta}{d} = \sin^{-1} \frac{1}{200}$
This is too close to the central maximum to b
desirable, but, because x-ray wavelengths are
such gratings cannot be constructed mechanic
In 1912, it occurred to German physicis
solid, wh is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is electrons levents a crease x-ray wavelengths are about equal to atomic diameters, accelerated gratings cannot be constructed mechanically. In This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is electrons leaves desirable, but, because x-ray wavelengths are about equal to atomic diameters, such grating cannot be constructed mechan desirable, but, because x-ray wavelengths are about equal to atomic diameters,
such gratings cannot be constructed mechanically.
In 1912, it occurred to German physicist Max von Laue that a crystalline
dimensional "diffra such gratings cannot be constructed mechanically.

In 1912, it occurred to German physicist Max von Laue that a crystalline

solid, which consists of a regular array of atoms, might form a natural three-

dimensional "dif dimensional "diffraction grating" for x rays. The idea is that, in a crystal such as
sodium chloride (NaCl), a basic unit of atoms (called the *unit cell)* repeats itself
throughout the array. Figure 36-28*a* represents a

This process of scattering and interference is a form of diffraction.

Fictional Planes. Although the process of diffraction of x rays by a crystal is

A different orientation of the incident x rays relative to the structure.A different family of parallel planes now effectively reflects the x rays.

Figure 36-27 X rays are generated when electrons leaving heated filament F are accelerated through a potential difference dow" W in the evacuated chamber C is transparent to x rays.

reflected by a family of parallel reflecting planes (or crystal planes) that extend through the atoms within the crystal and that contain regular arrays of the atoms. (The x rays are not actually reflecting *planes* (or *crystal planes*) that extend
through the atoms within the crystal and that contain regular arrays of the atoms.
(The x rays are not actually reflected; we use these fic plify the analysis of the actual diffraction process.)

Figure 36-28b shows three reflecting planes (part of a family containing many parallel planes) with *interplanar spacing d*, from which the incident rays reflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend
through the atoms within the crystal and that contain regular arrays of the atoms.
(The x rays are not actually reflected; we use thes Freflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend
through the atoms within the crystal and that contain regular arrays of the atoms.
(The x rays are not actually reflected; we use th **Solution**
Freflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend
through the atoms within the crystal and that contain regular arrays of the atoms.
(The x rays are not actually reflected defined relative to the *surface* of the reflecting plane rather than a normal to that reflected by a family of parallel *reflecting planes* (or *crystal planes*) that extend
through the atoms within the crystal and that contain regular arrays of the atoms.
(The x rays are not actually reflected; we use the equal to the unit cell dimension a_0 . . (The x rays are not actually reflected; we use these fictional planes only to sim-
plify the analysis of the actual diffraction process.)
Figure 36-286 shows three reflecting planes (part of a family containing
many paral plify the analysis of the actual diffraction process.)

Figure 36-28b shows three reflecting planes (part of a family containing

many parallel planes) with *interplanar spacing d*, from which the incident rays

shown are

Figure 36-28c shows an edge-on view of reflection from an adjacent pair of planes have been defined solely to explain the intensity maxima in the diffraction many parallel planes) with *interplanar spacing d*, from which the incident rays shown are said to reflect in A create relection the entiget. Rays 1, 2, and 3 reflect from the first, second, and third planes, respective shown are said to reflect. Rays 1, 2, and 3 reflect from the first, second, and third
planes, respectively. At each reflection the angle of incidence and the angle of re-
decina are presented with θ . Contrary to the cu the relative phase between the waves of rays 1 and 2 as they leave the crystal is flection are represented with θ . Contrary to the custom in optics, these angles are defined relative to the *surface* of the reflecting plane rather than a normal to that surface. For the situation of Fig. 36-28*b*, th length difference must be equal to an integer multiple of the wavelength λ of surface. For the situation of Fig. 36-28*b*, the
equal to the unit cell dimension a_0 .
Figure 36-28*c* shows an edge-on view o
planes. The waves of rays 1 and 2 arrive at
reflected, they must again be in phase beca
plan equal to the unit cell dimension a_0 .

Figure 36-28c shows an edge-on view of reflection from an adjacent pair of

planes. The waves of rays 1 and 2 arrive at the crystal in phase. After they are

reflected, they must a planes. The waves of rays 1 and 2 arrive at the crystal in phase. After they are reflected, they must again be in phase because the reflections and the reflecting planes have been defined solely to explain the intensity m reflected, they must again be in phase because the reflections and the r
planes have been defined solely to explain the intensity maxima in the di
of x rays by a crystal. Unlike light rays, the x rays do not refract upon e light rays, the x rays do not refract upon enterin
ot define an index of refraction for this situation.
The waves of rays 1 and 2 as they leave the crys
gth difference. For these rays to be in phase, the
equal to an int rays do not refract upon entering the

s of refraction for this situation. Thus,

ys 1 and 2 as they leave the crystal is

or these rays to be in phase, the path

ger multiple of the wavelength λ of

dashed perpendicul the relative phase between the waves of rays 1 and 2 as they leave the crystal is
set solely by their path length difference. For these rays to be in phase, the path
length difference must be equal to an integer multiple

$$
2d \sin \theta = m\lambda
$$
, for $m = 1, 2, 3, ...$ (Bragg's law), (36-34)

set solely by their path length difference. For these rays to be in phase, the path
length difference must be equal to an integer multiple of the wavelength λ of
the x rays.
Diffraction Equation. By drawing the dashe father shared the 1915 Nobel Prize in physics for their use of x rays to study the the x rays.
 Diffraction Equation. By drawing the dashed perpendiculars in Fig. 36-28*c*,

we find that the path length difference is 2*d* sin θ . In fact, this is true for any pair

of adjacent planes in the family o a Bragg angle. ind that the path length difference is 2d sin θ . In fact, this is true for any pair djacent planes in the family of planes represented in Fig. 36-28b. Thus, we e, as the criterion for intensity maxima for x-ray diffrac

ily of planes from which they can be said to reflect so that we can apply Bragg's have, as the criterion for intensity maxima for x-ray diffraction,
 $2d \sin \theta = m\lambda$, for $m = 1, 2, 3, ...$ (Bragg'slaw), (36-34)

where *m* is the order number of an intensity maximum. Equation 36-34 is called
 Bragg's law af 2d sin $\theta = m\lambda$, for $m = 1, 2, 3, ...$ (Bragg's law), (36-34)
where *m* is the order number of an intensity maximum. Equation 36-34 is called
Bragg's law after British physicist W. L. Bragg, who first derived it. (He and h 2d sin $\theta = m\lambda$, for $m = 1, 2, 3, ...$ (Bragg's law), (36-34)
where *m* is the order number of an intensity maximum. Equation 36-34 is called
Bragg's law after British physicist W. L. Bragg, who first derived it. (He and h where *m* is the order number of an intensity maximum. Equation 36-34 is called **Bragg's law** after British physicist W. L. Bragg, who first derived it. (He and his father shared the 1915 Nobel Prize in physics for their to explain the x-ray diffraction via Bragg's law. father shared the 1915 Nobel Prize in physics for their use of x ray
structures of crystals.) The angle of incidence and reflection in Eq. 3
a *Bragg angle.*
Regardless of the angle at which x rays enter a crystal, there Ex for their use of x rays to study the e and reflection in Eq. 36-34 is called

enter a crystal, there is always a fam-

reflect so that we can apply Bragg's

ucture has the same orientation as it

the beam enters the st structures of crystals.) The angle of incidence and reflection
a *Bragg angle.*
Regardless of the angle at which x rays enter a crystal,
ily of planes from which they can be said to reflect so that
law. In Fig. 36-28*d*, If yo planes from which they can be said to fellect so that we can apply Brag S
does in Fig. 36-28d, notice that the crystal structure has the same orientation as it
does in Fig. 36-28d, but the angle at which the beam en

Determining a Unit Cell. Figure 36-29 shows how the interplanar spacing d

$$
5d = \sqrt{\frac{5}{4}a_0^2},
$$

$$
d = \frac{a_0}{\sqrt{20}} = 0.2236a_0.
$$
 (36-35)

Figure 36-29 suggests how the dimensions of the unit cell can be found once the interplanar spacing has been measured by means of x-ray diffraction.

X-ray diffraction is a powerful tool for studying both x-ray spectra and the to explain the x-ray diffraction via Bragg's law.
 Determining a Unit Cell. Figure 36-29 shows how the interplanar spacing d

can be related to the unit cell dimension a_0 . For the particular family of planes

shown t **Determining a Unit Cell.** Figure 36-29 shows how the interplanar spacing d
can be related to the unit cell dimension a_0 . For the particular family of planes
shown there, the Pythagorean theorem gives
 $5d = \sqrt{\frac{5}{4}}a_0^$ can be related to the unit cell dimension a_0 . For the particular family of planes
shown there, the Pythagorean theorem gives
 $5d = \sqrt{\frac{2}{4}a_0^2}$, (36-35)
Figure 36-29 suggests how the dimensions of the unit cell can b another can then be used to determine the wavelength of radiation reaching it.The $5d = \sqrt{\frac{5}{4}a_0^2}$, (36-35)

Figure 36-29 suggests how the dimensions of the unit cell can be found once the

interplanar spacing has been measured by means of x-ray diffraction.

X-ray diffraction is a powerful tool fo only the spacing of various crystal planes but also the structure of the unit cell. Figure 36-29 A family of planes through the wavelengths at different angles. A detector that can discriminate one angle from

the edge length a_0 of a unit cell to the interplanar spacing d.

Review & Summary

Review & Summary
Diffraction When waves encounter an edge, an obstacle, or an **Double-Slit Diffraction** Wave
paperture the size of which is comparable to the wavelength of the each of width *a*, whose centers are
waves aperture the size of which is comparable to the wavelength of the each of width a, whose centers are a distance d apart, display dif-**Preview & Summary**
 Diffraction When waves encounter an edge, an obstacle, or an
 Double-Slit Diffraction

parture the size of which is comparable to the wavelength of the

waves, those waves spread out as they trave undergo interference.This is called diffraction.

Single-Slit Diffraction Waves passing through a long narrow **Solution Example 19:20**
 Summary
 Diffraction When waves encounter an edge, an obstacle, or an **Double-Slit Diffraction**

waves, those waves spread out as they travel and, as a result, fraction patterns w

undergo in **Pack of the Example 19**
 Diffraction When waves encounter an edge, an obstacle, or an
 Double-Slit Diffraction Waves waves, those waves spread out as they travel and, as a result, fraction patterns whose intensisty rated by minima located at angles θ to the central axis that satisfy **SUMMARY**

en waves encounter an edge, an obstacle, or an

f which is comparable to the wavelength of the

es spread out as they travel and, as a result,

ce. This is called **diffraction.**
 raction Waves passing through **12.** IV

Inter an edge, an obstacle, or an

arable to the wavelength of the

seach of width a, v

seasing through a long narrow

g screen, a **single-slit diffraction**

simum and other maxima, sepa-
 θ to the central a

$$
a \sin \theta = m\lambda
$$
, for $m = 1, 2, 3, ...$ (minima). (36-3)

The intensity of the diffraction pattern at any given angle θ is

$$
I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha}\right)^2, \text{ where } \alpha = \frac{\pi a}{\lambda} \sin \theta \qquad (36-5, 36-6)
$$

and I_m is the intensity at the center of the pattern.

Circular-Aperture Diffraction Diffraction by a circular aperture or a lens with diameter d produces a central maximum **and contract maximum concentric maximal concentral and concentral axis that satisfy**

and $\theta = m\lambda$, for $m = 1, 2, 3, ...$ (minima). (36-3) used to separate an

The intensity of the diffraction pattern at any given angle θ angle θ given by Fraction pattern at any given angle θ is

by N (multipude)
 $\left(\frac{36-5}{36-6}\right)$ d sin $\theta =$

with the half

the center of the pattern.
 Diffraction Diffraction by a circular

and minima, with the first minimum at a

$$
\sin \theta = 1.22 \frac{\lambda}{d} \quad \text{(first minimum—circular aperture)}.\tag{36-12}
$$

Rayleigh's Criterion Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular such as x rays. For analysis purposes, the atoms can be visualized as separation can then be no less than

$$
\theta_{\rm R} = 1.22 \frac{\lambda}{d} \qquad \text{(Rayleigh's criterion)}, \tag{36-14}
$$

passes.

Questions

1 You are conducting a single-slit diffraction experiment 4 For three experiments, Fig. 36-31 gives separation can then be no less than
 $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion), (36-14) Diffraction maxim

in which *d* is the diameter of the aperture through which the light

planes, and the wave

passes.

2*d* sin $\theta =$ $\theta_R = 1.22 \frac{\lambda}{d}$ (Rayleigh's criterion), (36-14) Diffraction maximi-
in which *d* is the diameter of the aperture through which the light
planes, and the wave
passes.
2*d* sin $\theta = m\lambda$,
Questions
1 You are conducting $\theta_R = 1.22 \frac{1}{d}$ (Rayleigh's criterion), (36-14) incident direction c
in which *d* is the diameter of the aperture through which the light
planes, and the wave
passes.
2d sin $\theta = m\lambda$,
2d sin $\theta = m\lambda$,
2d sin $\theta =$ (b) 4.5λ ? **2** 2 in $\theta = m$
 2 2 in a point at which the top and bottom rays through

the slit have a path length difference equa **1** You are conducting a single-slit diffraction experiment **4** For three experiment by the light of wavelength λ . What appears, on a distant viewing α versus angle θ in screen, at a point at which the top and bo **Questions**

1 You are conducting a single-slit diffraction experiment **4** For three experiment hight of wavelength λ . What appears, on a distant viewing α versus angle θ in screen, at a point at which the top an **1** You are conducting a single-slit diffraction experiment **4** I
with light of wavelength λ . What appears, on a distant viewing α ve
screen, at a point at which the top and bottom rays through
the slit have a path 1 You are conducting a single-slit diffraction experiment
with light of wavelength λ . What appears, on a distant viewing
screen, at a point at which the top and bottom rays through
the slit have a path length differenc

through the slit arrive at a certain point on the viewing screen with chain of phasors make? screen, at a point at which the top and bottom rays through
the slit have a path length difference equal to (a) 5 λ and
(b) 4.5 λ ?
2 In a single-slit diffraction experiment, the top and bottom rays
through the slit

 θ for two-slit interference using light of the slit have a path length difference equal to (a) 5 λ and perim

(b) 4.5 λ ? and and (
 2 In a single-slit diffraction experiment, the top and bottom rays

through the slit arrive at a certain point on the viewing (b) 4.5*x*?
 2 In a single-slit diffraction experiment, the top and bottom rays

through the slit arrive at a certain point on the viewing screen with

a path length difference of 4.0 wavelengths. In a phasor representa arations and (b) the total number of twoslit interference maxima in the pattern, greatest first.

Figure 36-30 Question 3.

Double-Slit Diffraction Waves passing through two slits, **COLUMERT ASSET CONSTRIMEL SET ASSET AND A THANGER ASSET AND A THANGER** fraction patterns whose intensity I at angle θ is

$$
I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2 \quad \text{(double slit)}, \tag{36-19}
$$

with $\beta = (\pi d/\lambda) \sin \theta$ and α as for single-slit diffraction.

Magnetics are

as a result, fraction patterns whose centers are

fraction patterns whose intensity I_i

ong narrow
 $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)$

ong narrow

with $\beta = (\pi d/\lambda) \sin \theta$ and α as for s

that satisfy
 Diffr Diffraction Gratings A diffraction grating is a series of "slits" used to separate an incident wave into its component wavelengths **Double-Slit Diffraction** Waves passing through two slits,
each of width a, whose centers are a distance d apart, display dif-
fraction patterns whose intensity I at angle θ is
 $I(\theta) = I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2$ (double s by N (multiple) slits results in maxima (lines) at angles θ such that h *a*, whose centers are a distance *d* apart, display dif-

erns whose intensity *I* at angle θ is
 θ) = $I_m(\cos^2 \beta) \left(\frac{\sin \alpha}{\alpha}\right)^2$ (double slit), (36-19)
 I/λ) sin θ and α as for single-slit diffraction.
 s are a distance *d* apart, display dif-
ity *I* at angle θ is
 $\frac{\sin \alpha}{\alpha}$ $\Big)^2$ (double slit), (36-19)
for single-slit diffraction.
liffraction grating is a series of "slits"
vave into its component wavelengths
th eries of "slits"

it wavelengths

it wavelengths

is θ such that

(36-25)

(36-28)

(36-29, 36-30)

(36-31, 36-32)

 $d \sin \theta = m\lambda$, for $m = 0, 1, 2, \dots$ (maxima),

with the **half-widths** of the lines given by

and

$$
\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta} \quad \text{(half-widths)}.\tag{36-28}
$$

The dispersion D and resolving power R are given by

$$
D = \frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta} \tag{36-29, 36-30}
$$

results in maxima (lines) at angles
$$
\theta
$$
 such that
\nfor $m = 0, 1, 2, ...$ (maxima), (36-25)
\nof the lines given by
\n
$$
= \frac{\lambda}{Nd \cos \theta}
$$
 (half-widths). (36-28)
\nd resolving power R are given by
\n
$$
D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}
$$
 (36-29, 36-30)
\n
$$
R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm.
$$
 (36-31, 36-32)
\n**n** The regular array of atoms in a crystal is a
\ndiffraction grating for short-wavelength waves

incident direction of the wave, measured from the surfaces of these in which d is the diameter of the aperture through which the light planes, and the wavelength λ of the radiation satisfy **Bragg's law: X-Ray Diffraction** The regular array of atoms in a crystal is a three-dimensional diffraction grating for short-wavelength waves $\Delta \theta_{\text{hw}} = \frac{\lambda}{Nd \cos \theta}$ (half-widths). (36-28)

The dispersion *D* and resolving power *R* are given by
 $D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$ (36-29, 36-30)

and
 $R = \frac{\lambda_{\text{avg}}}{\Delta \lambda} = Nm.$ (36-31, 36-32)
 X-Ray Diffraction The being arranged in planes with characteristic interplanar spacing d. Diffraction maxima (due to constructive interference) occur if the The dispersion *D* and resolving power *R* are given by
 $D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$ (36-29, 36-30)

and
 $R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm.$ (36-31, 36-32)
 X-Ray Diffraction The regular array of atoms in a crystal is a

three-dimen $D = \frac{\Delta \theta}{\Delta \lambda} = \frac{m}{d \cos \theta}$ (36-29, 36-30)
and
 $R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm.$ (36-31, 36-32)
X-Ray Diffraction The regular array of atoms in a crystal is a
three-dimensional diffraction grating for short-wavelength waves
such as $D = \frac{\lambda_{avg}}{\Delta \lambda} = \frac{d \cos \theta}{d \cos \theta}$ (36-29, 36-30)
 $R = \frac{\lambda_{avg}}{\Delta \lambda} = Nm.$ (36-31, 36-32)
 action The regular array of atoms in a crystal is

onal diffraction grating for short-wavelength wave

For analysis purposes, the ato $\frac{avg}{dA} = \frac{1}{d \cos \theta}$ (36-29, 36-30)
 $\frac{avg}{dA} = Nm.$ (36-31, 36-32)

egular array of atoms in a crystal is a

1 grating for short-wavelength waves

rrposes, the atoms can be visualized as

constructive interplanar spacin such as x rays. For analysis purposes, the atoms can be visualized as
being arranged in planes with characteristic interplanar spacing d.
Diffraction maxima (due to constructive interference) occur if the
incident directi

$$
2d \sin \theta = m\lambda
$$
, for $m = 1, 2, 3, ...$ (Bragg's law). (36-34)

Diffraction maxima (due to constructive interference) occur if the
incident direction of the wave, measured from the surfaces of these
planes, and the wavelength λ of the radiation satisfy **Bragg's law:**
2d sin $\theta = m\lambda$ periments according to (a) the slit widths and (b) the total number of diffraction minima in the pattern,greatest first. 4 For three experiments, Fig. 36-31 gives α
 α versus angle θ in one-slit diffraction using

light of wavelength 500 nm. Rank the ex-

periments according to (a) the slit widths

and (b) the total number of diffr 4 For three experiments, Fig. 36-31 gives α
 α versus angle θ in one-slit diffraction using

light of wavelength 500 nm. Rank the ex-

periments according to (a) the slit widths

and (b) the total number of diffr 4 For three experiments, Fig. 36-31 gives α
 α versus angle θ in one-slit diffraction using

light of wavelength 500 nm. Rank the ex-

periments according to (a) the slit widths

and (b) the total number of diffr The matter capacitation is rigg 50-51 given and the waves due to diffraction wing
tight of wavelength 500 nm. Rank the experiments according to (a) the slit widths
and (b) the total number of diffraction
minima in the pat

5 Figure 36-32 shows four choices for the rectangular opening of a source of either sound waves or light waves.The sides have

the openings according to the extent of (a) left-right spreading and (b) up-down spreading of the waves due to diffraction, greatest first.

Figure 36-31 Question 4. θ (rad)

 α

0 $\pi/2$

 \overline{A} B

 $C \qquad \qquad$

1108 CHAPTER 36 DIFFRACTION

6 Light of frequency f illuminating a long narrow slit produces a **1108** CHAPTER 36 DIFFRACTION
 6 Light of frequency f illuminating a long narrow slit produces a

diffraction pattern. (a) If we switch to light of frequency 1.3f, does

two orders produced by a single diffraction patte **1108** CHAPTER 36 DIFFRACTION
 6 Light of frequency filluminating a long narrow slit produces a

diffraction pattern (a) If we switch to light of frequency 1.3f, does

the pattern expand away from the center or contract merge the equipment in clear corn syrup?

7 At night many people see rings (called *entoptic halos*) surround-**1108** CHAPTER 36 DIFFRACTION
 6 Light of frequency filluminating a long narrow slit produces a

diffraction pattern. (a) If we switch to light of frequency 1.3f, does

two orders produced by a sing

the pattern expand are the first of the side maxima in diffraction patterns produced by structures that are thought to be within the cornea (or possibly the **1108** CHAPTER 36 DIFFRACTION
 6 Light of frequency *f* illuminating a long narrow slit produces a

diffraction pattern. (a) If we switch to light of frequency 1.3*f*, does

two orders

the pattern expand away from the overlap the lamp.) (a) Would a particular ring become smaller or larger if the lamp were switched from blue to red light? (b) If a lamp emits white light, is blue or red on the outside edge of the ring? the pattern expand away from the center or contract toward the

the pattern expand or contract if, instead, we sub-

innes, the left pair

merge the equipment in clear corn syrup?
 7 At night many people see rings (call center? (b) Does the pattern expand or contract if, instead, we sub-

ines, the left pair or right pair, are in

merge the equipment in clear corn syrup?
 7 At night many people see rings (called *entoptic halos*) surro 7 At night many people see rings (called *entoptic halos*) surround-

ing bright outdoor lamps in otherwise dark surroundings. The rings

are the first of the side maxima in diffraction patterns produced by

structures th

remain the same as the wavelength increases? (b) For a given order or in the third order? overlap the lamp.) (a) Would a particular ring becom
larger if the lamp were switched from blue to red light?
emits white light, is blue or red on the outside edge of the
8 (a) For a given diffraction grating, does the Larger if the lamp were switched from blue to red light? (b) If a lamp

emits white light, is blue or red on the outside edge of the ring?
 8 (a) For a given diffraction grating, does the smallest difference
 $\Delta \lambda$ in **8** (a) For a given diffraction grating, does the smallest difference
 8 (a) For a given diffraction grating, does the smallest difference
 $\Delta \lambda$ in two wavelengths that can be resolved increases, decrease, or

the cen **12** Figure

Al in two wavelengths that can be resolved increase, decrease, or

Al in two wavelength region (say, around 500 nm), is Δλ greater in the first

order or in the third order?

9 Figure 36-33 shows a red line a

9 Figure 36-33 shows a red line and a green line of the same order in the pattern produced by a diffraction

removing tape that had covered the outer half of the rulings—
would (a) the half-widths of the lines and (b) the separation of the separation connection is using field of view and their distances from Example the right, shift to the lines increase of the right of the right, and the shift to the right, shift to the right,

wavelength region (say, around 500 nm), is $\Delta \lambda$ greater in the first lope in two double-slift
order or in the third order?
 9 Figure 36-33 shows a red line and

a green line of the same order in the

pattern produced order or in the third order?
 9 Figure 36-33 shows a red line and

a green line of the same order in the

pattern produced by a diffraction
 Figure 36-33 Questions 9

of rulings in the grating and 10.

In three arra a 9 Figure 36-33 shows a red line and

a green line of the same order in the

pattern produced by a diffraction

of rulings in the grating—say, by

of rulings in the grating—say, by

and 10.

the slit separation d,

than, l (a) Rank the shift to the right,shift to the right,shift to the left, or remain the same of the same order and a shift to the right, shift to the left, or remain in place?
 Contains a share of rulings in the grating—say, grating. If we increased the number
of rulings in the grating—say, by
tremoving tape that had covered the outer half of the rulings—
next and $\frac{1}{3}$ In three arrangement
originals of the lines and $\left(\frac{1}{3}\right)$ and \left of rulings in the grating—say, by

of rulings in the grating—say, by

removing tape that had covered the outer half of the rulings—

would (a) the half-widths of the lines and (b) the separation of the

the objects that a

has the greater number of rulings? (b) Figure 36-34b shows lines of two orders produced by a single diffraction grating using light of has the greater number of rulings? (b) Figure 36-34*b* shows lines of
two orders produced by a single diffraction grating using light of
two wavelengths, both in the red region of the spectrum. Which
lines, the left pair has the greater number of rulings? (b) Figure 36-34b shows lines of
two orders produced by a single diffraction grating using light of
two wavelengths, both in the red region of the spectrum. Which
lines, the left pair or the center of the diffraction pattern located to the left or to the has the greater number of rulings? (b) Figure 36-34b shows lines of
two orders produced by a single diffraction grating using light of
two wavelengths, both in the red region of the spectrum. Which
lines, the left pair or the pattern expand away from the center or contract toward the two wavelengths, both in the red region of the spectrum. Which umber of rulings? (b) Figure 36-34*b* shows lines of
coed by a single diffraction grating using light of
both in the red region of the spectrum. Which
or right pair, are in the order with greater m ? Is
diffraction patte

12 Figure 36-35 shows the bright fringes that lie within \overrightarrow{A} bright fringes that lie within the central diffraction envelope in two double-slit diffraction experiments using the same wavelength of light.

Figure 36-33 Questions 9 the slit separation d, and (c) the ratio d/a in experiment B greater than, less than, or the same as those quantities in experiment A?

13 In three arrangements you view two closely spaced small 12 Figure 36-35 shows the

bright fringes that lie within A

the central diffraction enve-

lope in two double-slit dif-

fraction experiments using the same wavelength of light.

Figure 36-35 Question 12.

Are (a) the the objects occupy in your field of view and their distances from **Example 1** 3 and diffraction envelope in two double-slit diffraction envelope in two double-slit diffraction experiments using the same wavelength of light. **Figure 36-35** Question 12. Are (a) the slit width a, (b) the s (a) Rank the arrangements according to the separation between the central dimaction cirve-

lope in two double-slit dif-

fraction experiments using B

the same wavelength of light.

Are (a) the slit width a, (b)

the slit separation d, and (c) the ratio d/a in experiment B greate fraction experiments using B
fraction experiments using B
the same wavelength of light. **Figure 36-35** Question 12.
Are (a) the slit width a, (b)
the slit separation d, and (c) the ratio d/a in experiment B greater
th and (c) arrangement 3? Are (a) the slit width a, (b)
the slit separation d, and (c) the ratio d/a in experiment B greater
than, less than, or the same as those quantities in experiment A?
13 In three arrangements you view two closely spaced sma the slit separation d, and (c) the ratio d/a in experiment B greater than, less than, or the same as those quantities in experiment A?
 13 In three arrangements you view two closely spaced small objects that are the sam than, less than, or the same as those quantities in experiment A?

13 In three arrangements you view two closely spaced small

objects that are the same large distance from you. The angles that

the objects occupy in your

11 (a) Figure 36-34a shows the lines produced by diffraction ruling spacing is 1/3.5. Without written calculation or use of a calpear in the diffraction pattern.

Problems

Module 36-1 Single-Slit Diffraction

•1 **C** The distance between the first and fifth minima of a single-

single slit to have the first diffraction minimum at $\theta = 45.0^{\circ}$?

width of $a = 0.40$ mm. A thin converging lens of focal length $+70$ cm **Module 36-1 Single-Slit Diffraction**
 Columber 1 The distance between the first and fifth minima of a single-

slit diffraction pattern is 0.35 mm with the screen 40 cm away from

the signal diffraction pattern is 0.35 **Module 36-1 Single-Slit Diffraction**
 •1 CD The distance between the first and fifth minima of a single-

slit diffraction pattern is 0.35 mm with the screen 40 cm away from

the slit, when light of wavelength 550 nm i fraction pattern to the first minimum? slit diffraction pattern is 0.35 mm with the screen 40 cm away from
the slit, when light of wavelength 550 nm is used. (a) Find the slit
width. (b) Calculate the angle θ of the first diffraction minimum.
•2 What must the slit, when light of wavelength 550 nm is used. (a) Find the slit
width. (b) Calculate the angle θ of the first diffraction minimum.
from towers ha
single slit to have the first diffraction minimum at $\theta = 45.0^{\circ}$

•2 What must be the ratio of the slit width to the wavelength for a strategies in wavelength increase or decrease the diffraction of the $\frac{1}{2}$ 45.0°? signals into the shadow regions of obstacles? Assume that a **EXECUTE: EXECUTE: EXECUTE:** is placed between the slit and a viewing screen and focuses the fraction maximum (out to the first minima) for wavelengths of tion

ign

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Interactive solution is at

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tower because of a hill or building, it can still intercept a signal if

the signal ign

"Worked-out solution is at **http://www.wiley.com/college/halliday**

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tower because of a hill or building, it can still intercept a sig ign

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tower because of a hill or building, it can still intercept a signal if

the signal diffracts enough around the obst from the solution is at **http://www.wiley.com/college/halliday**

Interactive solution is at **http://www.wiley.com/college/halliday**

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tower because of a hill or building, it can still intercept a signal if
 change in wavelength increase or decrease the diffraction of the Worked-out solution is at **http://www.wiley.com/college/halliday**
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tower because of a hill or building, it can still intercept a signal if
the signal diffracts enough around the obstacle, i Interactive solution is at **http://www.wiley.com/college/halliday**
Isofphysics.com
Interactive solution of the signal diffracts enough around the obstacle, into the obstacle's
"shadow region." Previously, television signa **Example 10** soft
 Example 10 software of a hill or building, it can still intercept a signal if

the signal diffracts enough around the obstacle, into the obstacle's

"shadow region." Previously, television signals had tower because of a hill or building, it can still intercept a signal i
the signal diffracts enough around the obstacle, into the obstacle'
"shadow region." Previously, television signals had a wavelength o
about 50 cm, bu into the obstacte's
ad a wavelength of
t are transmitted
mm. (a) Did this
diffraction of the
? Assume that a
dth between two
of the central dif-
pr wavelengths of
lengths λ_a and λ_b ,
the λ_a component
component. (a form, the violarly, the vision signals had a waver
t 50 cm, but digital television signals that are tran
towers have a wavelength of about 10 mm. (a) 1
ge in wavelength increase or decrease the diffractio
ls into the shad

•5 A single slit is illuminated by light of wavelengths λ_a and λ_b , , chosen so that the first diffraction minimum of the λ_a component coincides with the second minimum of the λ_b component. (a) If $\lambda_b = 350$ nm, what is λ_a ? For what order number m_b (if any) does a

minimum of the λ_b component coincide with the minimum of the scale is set by $\alpha_s = 12$ rad. What are α (rad)
 λ_c component in the order number (b) $m_s = 2$ and (c) $m_s = 3$? (a) the slit width (b) the total num- λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$? (a) the s

minimum of the λ_b component coincide with the minimum of the
 λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$? (a) the slit width, (t)
 •6 Monochromatic light of wavelength 441 nm is incident on a ber minimum of the λ_b component coincide with the minimum of the scale is set by α_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$? (a) the slit width of **screen 2.00** m away, the distance between the second minimum of the λ_b component coincide with the minimum of the
 λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$? (a) the slit width, (b) the total num-
 •6 Monochromatic light of wavelength 441 nm is in (a) Calculate the angle of diffraction θ of the second minimum.
(b) Find the width of the slit mum, and (d) the greatest angle for minimum of the λ_b component coincide with th λ_a component in the order number (b) $m_a = 2$ an **6** Monochromatic light of wavelength 441 nr narrow slit. On a screen 2.00 m away, the distsecond diffraction minimum and minimum of the λ_{*b*} component coincide with the minimum of the scale is set by $\alpha_s = 12$ rad. What are λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$? (a) the slit width, (b) the total num-
 •6 Monoc λ_a component in the order number (b) $m_a = 2$ and (c) $m_a = 3$?

•**6** Monochromatic light of wavelength 441 nm is incident on a narrow slit. On a screen 2.00 m away, the distance between the second diffraction minimum an

angle between the first diffraction minimum on one side of the $\frac{13}{13}$ Monochromatic light with wavelength 538 nm is incident on a central maximum and the first minimum on the other side is 1.20° slit with width 0.025 What is the width of the slit?

•• 8 Sound waves with frequency through the rectangular opening of a speaker cabinet and into a large axis auditorium of length $d = 100$ m. The (a) Calculate the angle of diffraction θ of the second

(b) Find the width of the slit.

•7 Light of wavelength 633 nm is incident on a narro

angle between the first diffraction minimum on one

central maximum and the (b) Find the width of the slit.
 a minim angle between the first diffraction minimum on one side of the

angle between the first diffraction minimum on one side of the

central maximum and the first minimum on the other **7** Light of wavelength 633 nm is incident on a narrow slit. T
angle between the first diffraction minimum on one side of t
central maximum and the first minimum on the other side is 1.20
What is the width of the slit?
8 how far from the central axis will a •• 8 Sound waves with frequency

3000 Hz and speed 343 m/s diffract

through the rectangular opening of

a speaker cabinet and into a large

auditorium of length $d = 100$ m. The

opening, which has a horizontal

width of

Figure 36-36 Problem 8.

listener be at the first diffraction minimum and thus have difficulty hearing the sound? (Neglect reflections.)

away. What is the distance between the first two diffraction minima points in the pattern where the intensity is one-half that at the center
on the same side of the central diffraction maximum? of the pattern. (See Fig. 36 on the same side of the central diffraction maximum? •• 9 SSM ILW A slit 1.00 mm wide is illuminated by light of wave-

dimension) sometimes use a laser to continually monitor the width of 30.0 cm, faces a wall 100 m

width of 30.0 cm, faces a wall 100 m

he center of the pattern. Also let I_P represent the into

how far from the centeral axis will a

line center of the pattern? (b) Determ

listen ducing a diffraction pattern like that of a single slit of the same
ducing a diffraction pattern like that of a single slit of the same
 $(d) 1.00\lambda$, (e) 5.00λ , and (f) 10.0λ . how far from the central axis will a

listener be at the first diffraction minimum and thus have difficulty
 hereaving the solution from the signal with A slit 1.00 mm wide is illuminated by light of wave-
 extraction listener be at the first diffraction minimum and thus have difficulty
 EVALUA A slit 1.00 mm wide is illuminated by light of wave-
 EVALUA A slit 1.00 mm wide is illuminated by light of wave-
 EVALUA A slit 1.00 mm tion pattern appears on a screen at distance $L = 2.60$ m. If the **••9 SSM ILW** A slit 1.00 mm wide is illuminated by light of wave-

length 589 nm. We see a diffraction pattern on a screen 3.00 m central diffraction

away. What is the distance between the first two diffraction minima
 between the two tenth-order minima (one on each side of the central maximum)?

Module 36-2 Intensity in Single-Slit Diffraction

•11 A 0.10-mm-wide slit is illuminated by light of wavelength **Figure 36-37** Problem 10.
 Example 19 Michael State Concepts, shown that the value of α at which intensity at *P* is identical for the slite of the slit is illuminated by light of wavelength and *t* and *B* is viewe Wire

He-Ne

laser

laser Huygens wavelets arriving at point P from the top and midpoint of
the electric Check and midpoint of the state of m associated with the maxima in the sugge-
the stitute Sec. Because 264) **Example 19** He-Ne
 Example 18-37 Problem 10.
 Module 36-2 Intensity in Single-Slit Diffraction

11 A 0.10-mm-wide slit is illuminated by light of wavelength

589 nm. Consider a point P on a viewing screen on which th **Experiment 17** (a) Show that the Figure 36-37 Problem 10.
 Module 36-2 Intensity in Single-Slit Diffraction

11 A 0.10-mm-wide slit is illuminated by light of wavelength diffraction

11 A 0.10-mm-wide slit is illuminat

scale is set by $\alpha_s = 12$ rad. What are PROBLEMS 1109

scale is set by $\alpha_s = 12$ rad. What are α (rad)

(a) the slit width, (b) the total num-

ber of diffraction minima in the pat-

tern (count them on both sides of

the center of the diffraction patber of diffraction minima in the pattern (count them on both sides of the center of the diffraction pat-**EXECUTE:**

Scale is set by $\alpha_s = 12$ rad. What are α_s (rad)

(a) the slit width, (b) the total number of diffraction minima in the pat-

tern (count them on both sides of

the center of the diffraction pat-

tern), (c **EXECUTE SET ASSAUTE SET ASSAUTE SCALL SET ASSAUTE SCALL SCALL SCALL SET AND SET AND THE GREATER (COUNT them on both sides of the center of the diffraction pattern), (c) the least angle for a minimum, and (d) the greatest** a minimum? (a) the slit width, (b) the total num-

EXECUTE MS

Scale is set by $\alpha_s = 12$ rad. What are α_s (rad)

(a) the slit width, (b) the total num-

ber of diffraction minima in the pat-

tern (count them on both sides of

the center of the diffraction pat-

tern scale is set by $\alpha_s = 12$ rad. What are α (rad)

(a) the slit width, (b) the total num-

ber of diffraction minima in the pat-

tern (count them on both sides of

the center of the diffraction pat-

tern), (c) the leas scale is set by $\alpha_s = 12$ rad. What are α_s (rad)

(a) the slit width, (b) the total num-

ber of diffraction minima in the pat-

tern (count them on both sides of

the center of the diffraction pat-

tern), (c) the lea that point to the intensity at the central maximum. ber of diffraction minima in the pat-
tern (count them on both sides of
the center of the diffraction pat-
tern), (c) the least angle for a mini-
mum, and (d) the greatest angle for
a minimum?
Figure 36-38 Problem 12.
 central maximum and the first minimum on the other side is 1.20° . slit with width 0.025 mm. The distance from the slit to a screen is 3.5
What is the width of the slit?
What is the width of the slit?

tern (count them on both sides of
the center of the diffraction pat-
tern), (c) the least angle for a mini-
mum, and (d) the greatest angle for
a minimum?
Figure 36-38 Problem 12.
13 Monochromatic light with wavelengt ing screen be at distance $D = 3.00$ m. Let a y axis extend upward pat-

inini-

le for
 $\begin{array}{c} 0 & 0.5 & 1 \\ \end{array}$ sin θ
 Figure 36-38 Problem 12.

wavelength 538 nm is incident on a

stance from the slit to a screen is 3.5

n 1.1 cm from the central maximum.
 α , and (c) the rati tern), (c) the least angle for a mini-
mum, and (d) the greatest angle for
a minimum?
Figure 36-38 Problem 12.
Talon Alon Constant is of the viewer of the solution of the situation of a
situation with width 0.025 mm. mum, and (d) the greatest angle for 0.5 and 0.5 and μ and μ point P at $y = 15.0$ cm. (a) What is the ratio of I_p to the intensity I_m at **Figure 36-38** Problem 12.

comatic light with wavelength 538 nm is incident on a

0.025 mm. The distance from the slit to a screen is 3.5

point on the screen 1.1 cm from the central maximum.

for that point, (b) α , a the center of the pattern? (b) Determine where point P is in the diffraction pattern by giving the maximum and minimum between Solution 10.025 mm. The ustained from the sit to a series
m. Consider a point on the screen 1.1 cm from the central maximal
Calculate (a) θ for that point, (b) α , and (c) the ratio of the inten
that point to the int that point to the intensity at the central maximum.
 •14 In the single-slit diffraction experiment of Fig. 36-4, let the wave-

length of the light be 500 nm, the slit width be 6.00 μ m, and the view-

ing screen be a **Proper and the light be 500 nm, the slit width be 6.00** μ **m, and the view-
ling screen be at distance** $D = 3.00$ **m. Let a y axis extend upward
along the viewing screen, with its origin at the center of the diffraction
pa Example 189** and wave with interaction pattern. (See Fig. 36-36). (a) Suppose a diffraction pattern of length 589 nm, we see a diffraction pattern on a screen 3.00 m
 Example 58 nm. and thus a horizontal width of 30.0 c Central \bullet 14 In the single-slit diffraction experiment of Fig. 36-4, let the wavelength of the light be 500 nm, the slit width be 6.00 μ m, and the viewpattern. Also let I_p represent the intensity of the diffracted light at

••15 SSM WWW The full width at half-maximum (FWHM) of a points in the pattern where the intensity is one-half that at the center one-half the maximum value when $\sin^2 \alpha = \alpha^2/2$. (b) Verify that $\alpha =$ ω , ω and the view-
 ω axis extend upward

the diffraction

the diffracted light at
 ω to the intensity I_m at
 ω e point *P* is in the dif-

d minimum between

t lies.

simum (FWHM) of a

angle between the Ing streen be at ustance $D = 3.00$ in. Let a y axis extend upward
along the viewing screen, with its origin at the center of the diffraction
pattern. Also let I_p represent the intensity of the diffracted light at
point along the viewing screen, which is origin at the center of the
pattern. Also let I_P represent the intensity of the diffract
point P at $y = 15.0$ cm. (a) What is the ratio of I_P to the in
the center of the pattern? (b) (a). (c) Show that the FWHM is $\Delta \theta = 2 \sin^{-1}(0.443 \lambda/a)$, where a is the (b) the diffracted light at f_{P} to the intensity I_{m} at ere point *P* is in the dif-
and minimum between hit lies.
aximum (FWHM) of a e angle between the two
ne-half that at the center
at the intensity drops to $\alpha^{$ pation. Also identify the intensity of the time transity of the intensity I_m at the center of the pattern? (b) Determine where point *P* is in the diffraction pattern by giving the maximum and minimum between which it l point *T* at $y = 13.0$ cm. (a) what is the ratio of I_P to the intensity I_m at the center of the pattern? (b) Determine where point *P* is in the diffraction pattern by giving the maximum and minimum between which it l contrained diffraction maximum is defined as the angle bet
points in the pattern where the intensity is one-half that
of the pattern. (See Fig. 36-8b.) (a) Show that the inter-
one-half the maximum value when $\sin^2 \alpha = \alpha^2$ one-half the maximum value when sin² α = $\alpha^2/2$. (b) Verify that α = 0.9-20. (a) solution to the transcendental equation of (a). (c) Show that the FWHM is $\Delta \theta = 2 \sin^{-1}(0.443 \lambda/a)$, where *a* is the slit width. Ca ••10 **Co** Manufacturers of wire (and other objects of small 1.39 rad (about 80°) is a solution to the transcendental equation of which it lies, or the two minima betw

light of wave-
 15 SSM WWW The full width

screen 3.00 m

central diffraction maximum is defi

points in the pattern where the inte

of the pattern. (See Fig. 36-8*b.*) (a)

ects o

monochromatic beam of parallel light is incident on a "colli-Point P lies in the geometrical shadow region on a distant one-han the maximum value when sin $\alpha = \alpha/2$. (b) verty that it.

1.39 rad (about 80°) is a solution to the transcendental equation

(a). (c) Show that the FWHM is $\Delta \theta = 2 \sin^{-1}(0.443\lambda/a)$, where a is

slit width. Calculat 1.39 Tad (about 60) is a solution to the transic
contract (a). (c) Show that the FWHM is $\Delta \theta = 2 \sin^{-1}(0.443 \lambda/a)$, slit width. Calculate the FWHM of the central maximum
(d) 1.00 λ , (e) 5.00 λ , and (f) 10.0 λ .
 16 an opaque circle with a hole in (a) 1.000, (c) 3.000, and (1) 10.00.
 and B *Babinet's* principle. A

monochromatic beam of paral-

lel light is incident on a "colli-

mating" hole of diameter $x \ge \lambda$.

Point *P* lies in the geometrical

shadow region **16** *Babinet's principle.* A

monochromatic beam of paral-

lel light is incident on a "colli-

mating" hole of diameter $x \ge \lambda$.

Point *P* lies in the geometrical

shadow region on a *distant*

screen (Fig. 36-39*a*) monochromatic beam of paral-

lel light is incident on a "colli-

mating" hole of diameter $x \ge \lambda$.

Point *P* lies in the geometrical

shadow region on a *distant*

screen (Fig. 36-39*a*). Two dif-

fracting objects, sho intensity at P is identical for the two diffracting objects A and B. mating" hole of diameter $x \ge \lambda$. Wire-making \qquad the collimating hole. Object A is

••17 (a) Show that the values of α at which intensity maxima

for single-slit diffraction occur can be found exactly by differentiat

at a subset of the collimating hole. Object A is

an opaque circle with a hole in

it, and B is the "photographic

megative" of A. Using superpo-

sition concepts, show that the

intensity at P is identical for the

t taining the condition tan $\alpha = \alpha$. To find values of α satisfying this The commandig mole. Object A is

an opaque circle with a hole in

it, and B is the "photographic

megative" of A. Using superpo-

sition concepts, show that the

intensity at P is identical for the

two diffracting object tan α and the straight line $y = \alpha$ and an opaque circle with a note in

it, and B is the "photographic

it, and B is the "photographic

intensity at P is identical for the

two diffracting objects A and B.
 a17 (a) Show that the values

of α at which inte it, and *D* is the photographic megative" of *A*. Using superposition concepts, show that the intensity at *P* is identical for the two diffracting objects *A* and *B*. (b)
 and *a* two diffracting objects *A* and *B*. termine the values of m associated with the maxima in the singleslit particular the two diffracting objects A and B.

intensity at P is identical for the
 two diffracting objects A and B.
 end at which intensity maxima

for single-slit diffraction occur can be found exactly by dif maxima do not lie exactly halfway between minima.) What are the ***17** (a) Show that the values Figure 36-39 Problem 16.
 ***17** (a) Show that the values Figure 36-39 Problem 16.

of α at which intensity maxima

for single-slit diffraction occur can be found exactly by differenti-

a **•17** (a) Show that the values Figure 36-39 Problem 16.
of α at which intensity maxima
for single-slit diffraction occur can be found exactly by differenti-
ating Eq. 36-5 with respect to α and equating the result t $\alpha = (m + \frac{1}{2})\pi$, de-•12 Figure 36-38 gives α versus the sine of the angle θ in a single-slit dif-
(b) smallest α and (c) associated m, the (d) second smallest α and α

1110 CHAPTER 36 DIFFRACTION

Module 36-3 Diffraction by a Circular Aperture

 CHAPTER 36 DIFFRACTION
 Module 36-3 Diffraction by a Circular Aperture
 24 Let Entoptic halos. If someon
 18 The wall of a large room is covered with acoustic tile in in otherwise dark surroundings, the

whic far can a person be from such a tile and still distinguish the indi- CHAPTER 36 DIFFRACTION
 Module 36-3 Diffraction by a Circular Aperture
 18 The wall of a large room is covered with acoustic tile in in otherwise dark

which small holes are drilled 5.0 mm from center to center CHAPTER 36 DIFFRACTION
 Module 36-3 Diffraction by a Circular Aperture
 18 The wall of a large room is covered with acoustic tile in in otherwise dark surroundings

which small holes are drilled 5.0 mm from cen CHAPTER 36 DIFFRACTION
 Module 36-3 Diffraction by a Circular Aperture
 Module 36-3 Diffraction by a Circular Aperture
 18 The wall of a large room is covered with acoustic

which small holes are drilled 5.0

yourself just at the limit of resolving the grains if your pupil diame-**1110** CHAPTER 36 DIFFRACTION
 1110 CHAPTER 36 DIFFRACTION
 18 The wall of a large room is covered with acoustic tile in in otherwise dark surroundings, the

which small holes are drilled 5.0 mm from center to center. **Module 36-3 Diffraction by a Circular Aperture**
 Call 18 The wall of a large room is covered with acoustic tile in in otherwise dark

which small holes are drilled 5.0 mm from center to center. How

by bright and dar
 Module 36-3 Diffraction by a Circular Aperture
 e18 The wall of a large room is covered with acoustic tile in in otherwise dark surrounding

which small holes are drilled 5.0 mm from center to center. How

by bright a answer to (a) be larger or smaller? far can a person be from such a tile and still distinguish the indi-

braction pattern as in Fig. 36-10, with

vidual holes, assuming ideal conditions, the pupil diameter of the

observer's eye to be 4.0 mm, and the wavel Frange of 6.2 km, what is the pupil diameter of the

observer's eye to be 4.0 mm, and the wavelength of the room

light to be 550 nm?
 a light to be 550 nm?
 a light to be 550 nm?
 a light of resolving the grains i

•20 The radar system of a navy cruiser transmits at a wavelength 10^5 km. Assume a wavelength of 550 nm for the light. can be from each other and still be resolved as two separate objects Google Earth), and the telescopes on military surveillance satelby the radar system?

•21 SSM WWW Estimate the linear separation of two objects on Assume first that Mars that can just be resolved under ideal conditions by an observer on Earth (a) using the naked eye and (b) using the 200 in. $(= 5.1 \text{ m})$ Mount Palomar telescope. Use the following data: 1.1 m) From the grains has wavelength 650 nm? (b) If the grains were

2.1 and the light from them had wavelength 400 nm, would the

2.1 and the light from them had wavelength 400 nm, would the

2.1 and the light Prom tele distance to Mars = 8.0×10^7 km, diameter of pupil = 5.0 mm, by them had wavelength 400 nm, would the

108 hount Particular Tor smaller?

In of a navy cruiser transmits at a wavelength

10⁵ km. As

ular antenna with a diameter of 2.3 m. At a

is the smallest distance that two spe wavelength of light $= 550$ nm. For smaller?

In of a navy cruiser transmits at a wavelength

ular antenna with a diameter of 2.3 m. At a

is the smallest distance that two speedboats

r and still be resolved as two separate objects

imate the linear se

•22 Assume that Rayleigh's criterion gives the limit of resorange of 6.2 km, what is the smallest distance that two speedboats

can resolve objects on the

by the radar system?
 Pack is the space start of the summer of two objects on the
 Pack is the space shuttle and the inear can be from each other and still be resolved as two separate objects

by the radar system?
 Example 185M WWW Estimate the linear separation of two objects on

Mars that can just be resolved under ideal conditions by a **EXECTE FOR THE ASSOCTET CONTROLLET ASSOCTET (SCEPT ASSOCTED)**
 EXECUTED ASSOCTED
 EXECUTED: THE ASSOCTED (FOR THE ASSOCTED AND THE ASSOCTED (= 5.1 m) Mount Palomar telescope. Use the following data: $(= 5.1 \text{ m})$ Mou **21 SM** WWW Estimate the linear separation of two objects on

Mars that can just be resolved under ideal conditions by an

Mostever on Earth (a) using the naked eye and (b) using the 200 in.

(= 5.1 m) Mount Palomar teles **Example 1 EXELUSE TO A THE SET AND THE SET AND THE SET AND THE ANTIFY (C) UNITELNATE AND A THE SET AND A THE SET AND MULTIMET AND A THE SET AND MULTIMET AND MULTIMET AND MULTIMET AND MULTIMET AND MULTIMET AND MULTIMET** Muss that can just be resolved under ideat conditions by an expected under the solved under than 19 mosphere. Also assume than $(= 5.1 \text{ m})$ Mount Palomar telescope. Use the following data: $= 400 \text{ km}$ and that the wave top, and 8 m in heighted in the matter of pupil = 5.0 mm, the sphere. The set of the following data: (40 km) Mm in Palomar telescope. Use the following data: 400 km and that height = 550 nm, (a) 85 cm resolution of

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Figure 36-40 Problem 22.The Great Wall of China.

•23 SSM The two headlights of an approaching automobile are tance will the eye resolve them? Assume that the pupil diameter is SAP/Wide World Photos

Figure 36-40 Problem 22. The Great Wall of China.

Figure 36-40 Problem 22. The Great Wall of China.

1.4 m apart. At what (a) angular separation and (b) maximum distance will the eye resolve them? A that diffraction effects alone limit the resolution so that Rayleigh's criterion can be applied.

•18 The wall of a large room is covered with acoustic tile in in otherwise dark surroundings, the lamp appears to be surrounded •19 (a) How far from grains of red sand must you be to position subtends angular diameter 2.5° in the observer's view, what is the •24 Entoptic halos. If someone looks at a bright outdoor lamp **24** Entoptic halos. If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction pattern by bright and dark rings (hence halos) that are actually a circular dif-**Example 19.1** Entoptic halos. If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fractio **P24 THE Entoptic halos.** If someone looks at a bright outdoor lamp in otherwise dark surroundings, the lamp appears to be surrounded by bright and dark rings (hence *halos*) that are actually a circular diffraction patte **Example 12** Entoptic halos. If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction **24 Example 10** Entoptic halos. If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fracti **Example 12.5°** Entoptic halos If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fractio (linear) diameter of the structure producing the diffraction? **Example 124 Example 10** is someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction pa **24** Entoptic halos If someone looks at a bright outdoor lamp
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction pattern a Earth the dat surroundings, the lamp appears to be surrounded
in otherwise dark surroundings, the lamp appears to be surrounded
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction pattern a In our was can sarolon angly an any appears to be sarroundly
by bright and dark rings (hence *halos*) that are actually a circular dif-
fraction pattern as in Fig. 36-10, with the central maximum overlap-
ping the direct ping the direct light from the lamp. The diffraction is produced by
structures within the cornea or lens of the eye (hence *entoptic*). If the
lamp is monochromatic at wavelength 550 nm and the first dark ring
subtends an

25 ILW Find the separation of two points on the Moon's surface that can just be resolved by the 200 in. $(= 5.1 \text{ m})$ telescope at

bound the

Mount Palomar, assuming that this

diffraction effects. The distance fro

relength
 10^5 km. Assume a wavelength of 550

m. At a
 26 The telescopes on some com

caboats

can resolve objects on the ground

G **EXECUTE:** Assume that Nayleigh's enterion gives the film of reso-
lution of an astronaut's eye looking down on Earth's surface from a aperture diameter of the Hubble Space Telescope is 2.4 m, what •26 The telescopes on some commercial surveillance satellites Surveillance swithin the cornea or lens of the eye (hence *entoptic*). If the lamp is monochromatic at wavelength 550 nm and the first dark ring subtends angular diameter 2.5° in the observer's view, what is the (lin lamp is monochromatic at wavelength 550 nm and the first dark ring
subtends angular diameter 2.5° in the observer's view, what is the
(linear) diameter of the structure producing the diffraction?
***25 ILW** Find the separa Assume first that object resolution is determined entirely by Rayleigh's criterion and is not degraded by turbulence in the atmosphere.Also assume that the satellites are at a typical altitude of **Example 10** km and is vegalation of two points on the woold is determined by diffraction effects. The distance from Earth to the Moon is 3.8×10^5 km. Assume a wavelength of 550 nm for the light.
 ••26 The telesco would be the required diameter of the telescope aperture for wount a about an existing that thus separator is occurrented by
diffraction effects. The distance from Earth to the Moon is 3.8 \times
10⁵ km. Assume a wavelength of 550 nm for the light.
26 The telescopes on some comme ing that turbulence is certain to degrade resolution and that the **EXECTS** The telescopes on some commercial surveillance satellites
can resolve objects on the ground as small as 85 cm across (see
Grogle Earth), and the telescopes on military surveillance satel-
lites reportedly can res can you say about the answer to (b) and about how the military surveillance resolutions are accomplished? Figure 2.110, and unclease the mind y surventure satellities reportedly can resolve objects as small as 10 cm across.
Assume first that object resolution is determined entirely by Rayleigh's criterion and is not degraded Assume first that object resolution is determined entirely by
Assume first that object resolution is determined entirely by
Rayleigh's criterion and is not degraded by turbulence in the at-
mosphere. Also assume that the Exsume that und optic its bound is to degraded by turbulence in the at-
mosphere. Also assume that the satellites are at a typical altitude of
400 km and that the wavelength of visible light is 550 nm. What
would be the r Example 1. The wind at the satellities are at a typical altitude of
mosphere. Also assume that the satellities are at a typical altitude of
would be the required diameter of the telescope aperture for
(a) 85 cm resolution 400 km and that the wavelength of visible light is 550 nm. What
would be the required diameter of the telescope aperture for
(a) 85 cm resolution and (b) 10 cm resolution? (c) Now, consider-
ing that turbulence is certain would be the required diameter of the telescope aperture for

(a) 85 cm resolution and (b) 10 cm resolution? (c) Now, consider-

ing that turbulence is certain to degrade resolution and that the

aperture diameter of the

top, and 8 m in height? (c) Would the astronaut be able to resolve $\frac{28}{28}$. The wings of tiger beetles (Fig. 36-41) are colored by interference due to thin cuticle-like layers. In addition, these layers are arranged in patches that are 60 μ m across and produce difing that turbulence is certain to degrade resolution and that the aperture diameter of the Hubble Space Telescope is 2.4 m, what can you say about the answer to (b) and about how the military surveillance resolutions are aperture diameter of the Hubble Space Telescope is 2.4 m, what
can you say about the answer to (b) and about how the military
surveillance resolutions are accomplished?
27 If Superman really had x-ray vision at 0.10 nm

Figure 36-41 Problem 28.Tiger beetles are colored by pointillistic mixtures of thin-film interference colors. Kjell B. Sandved/Bruce Coleman, Inc./Photoshot Holdings Ltd.

what viewing distance from a wing puts you at the limit of resolv-What viewing distance from a wing puts you at the limit of resolv-

ing the different colored patches according to Rayleigh's criterion?

Use 550 nm as the wavelength of light and 3.00 mm as the diame-

emits light is the ter of your pupil.

•• 29 (a) What is the angular separation of two stars if their images are barely resolved by the Thaw refracting telescope at the away from the beam source. (b) What is the ratio of the beam inwhat viewing distance from a wing puts you at the limit of resolv-

ing the different colored patches according to Rayleigh's criterion?

Use 550 mm as the diameter of your pupil.

Lens diameter of your pupil.
 COM the what viewing distance from a wing puts you at the limit of resolv-
ing the different colored patches according to Rayleigh's criterion?
Use 550 nm as the wavelength of light and 3.00 mm as the diame-
ter of your pupil.
• tance between these barely resolved stars if each of them is 10 what viewing distance from a wing puts you at the limit of resolv-

ing the different colored patches according to Rayleigh's criterion? diffraction, with res

Use 550 nm as the wavelength of light and 3.00 mm as the diam what viewing distance from a wing puts you at the limit of resolv-
ing the different colored patches according to Rayleigh's criterion?
Use 550 nm as the wavelength of light and 3.00 mm as the diame-
ter of your pupil.
• what viewing distance from a wing puts you at the limit of resolv-
ing the different colored patches according to Rayleigh's criterion?
Use 550 nm as the wavelength of light and 3.00 mm as the diame-
ter of your pupil.
• What versus the different colored patches according to Rayleigh's criterion? Use 550 nm as the wavelength of light and 3.00 mm as the diame-
Use 550 nm as the wavelength of light and 3.00 mm as the diame-
 COM \bullet 29 (the image is associated entirely with diffraction at the lens aperture and not with lens shares are barely resolved by the Thaw refracting telescope at the Allegheny Observatory in Pittsburgh? The lens diameter is 76 cm a ture and not with lens "errors." **Example 12** and is the angular separation of two stars in then the singless are barely resolved by the Thaw refracting telescope at the Allegheny Observatory in Pittsburgh? The lens diameter is 76 cm and its focal length all depends the time in the vitreous in the vitreous humor that fills most of your except at the distring threat that the distring threat fills the share $\lambda = 550$ nm. (b) Find the distring from space, so neg tance betwee A pind it is detected to the diffraction of the diffraction pattern. It was a circular this telescope, find the diameter of the first dark ring in the diffraction pattern as a circular this telescope, find the diameter of

somewhat linguis is than Assume $A = 30$ and its boxid rules the secondary maxima from the diffraction pattern, so measured on a photographic plate placed at the this telescope, find the diameter of the first dark ring in that the defect of the diffraction as a circular of the diffraction partern as a circular bolo of this telescope, find the diameter of the first dark ring in the diffraction partern as a circular bolo of this telescope le circle of the first minimum in the diffraction pattern appear to For the size in your spectrum of the theorem size in the same that the structure of you intercept the image is associated entirely with diffraction at the lens aper-
the image is associated entirely with diffraction at th be image is associated entirely with diffraction at the lens aper-

the image is associated entirely with diffraction at the lens aper-

ture and not with lens "errors."
 a obtright, featureless background are diffracti diameter D' on the retina at distance $L' = 2.0$ cm from the front of the magne is associated in the state of the eye last the two functions white region that sum and not with lens "errors." White region that sum the vitreous humor that fills most of your eye. Sighting through too faint to **side of the execution** and not which case corrects.
 are the example the example of the example sharpens the diffraction patterns of defects to too in

a pinhole sharpens the diffraction pattern. If you also view a

sm ble light is $\lambda = 550$ nm. If the dot has diameter $D = 2.0$ mm and is E Floaters. The floaters you see when viewing a

ess background are diffraction patterns of defects

two faint, cold

too faint, to be

pens the diffraction pattern. If you also view a

ting is on the c

ing is on the dif distance $L = 45.0$ cm from the eye and the defect is $x = 6.0$ mm in in the vitreous humor that fills most of your eye. Sighting through

a pinhole sharpens the diffraction pattern. If you also view a

small circular dot, you can approximate the defect's size. Assume

somewhat larges

dot' Example the secondary maxima from the

Retina Retina

•• 31 SSM Millimeter-wave radar generates a narrower beam than **Example 19 Contract Control of the central maximum, from first minimum, produced by a 220 GHz radar** diameter circular antenna. (The frequency is chosen to coincide with a low-absorption atmospheric "window.") (b) What is 2θ for a **Example 12**

(a)
 Figure 36-42 Problem 30.
 Figure 36-42 Problem 30.
 Conventional microwave radar generates a narrower beam than
 Conventional microwave radar, making it less vulnerable to anti-

radar missiles (a)
 and 1.6 cm? (b)
 Figure 36-42 Problem 30.
 action 30.
 action 30.
 action 30.
 action 3.0 cm
 action 2.6 cm
 action 2.6 cm
 action 3.6 cm
 action 3.220 GHz radar beam emitted by a 55.0-cm

diamete **••31 SSM** Millimeter-wave radar generates a narrower beam than
conventional microwave radar, making it less vulnerable to anti-
radar missiles than conventional radar. (a) Calculate the angular
width 2*θ* of the central **••31 SSM** Millimeter-wave radar generates a narrower beam than conventional microwave radar, making it less vulnerable to anti-
radar missiles than conventional radar. (a) Calculate the angular width 2 θ of the central **•31 SSM** Millimeter-wave radar generates a narrower beam than

conventional microwave radar, making it less vulnerable to anti-

radar missiles than conventional radar. (a) Calculate the angular

width 2 θ of the centr

uted as the diffraction pattern of a circular hole whose diameter equals that of the diaphragm.Take the speed of sound in water to be and missies that Contentional radiat. (a) Cadculate the angluar since the angluar since the angluar since the since the since the since the normal to first minimum, produced by a 220 GHz radar beam emitted by a 55.0-cm-
d when the current in an interest in the diaphragm to the first minimum of the diaphragm to the first minimum.

when a low-absorption atmospheric "window.") (b) What is 2 θ for a

more conventional circular antenna that h minimum for a zaro Given and example in thin the diappear of 2.3 m

diameter circular antenna. (The frequency is chosen to coincide

with a low-absorption atmospheric "window.") (b) What is 2 θ for a

more conventional more conventional circular antenna that has a diameter of 2.3 m

and emits at wavelength 1.6 cm?
 Figure 36-43 Problem 34. The corona and
 Figure 36-43 Problem 34. The corona and

frequency of 25 kHz as an underwater s

•• 33 GO Nuclear-pumped x-ray lasers are seen as a possible

ing the different colored patches according to Rayleigh's criterion? be valid in the limit of resolv-

g to Rayleigh's criterion?

diffraction, with resulting did 3.00 mm as the diame-

a laser operating at a wave emits light is the end of

two stars if their im-

fracting telescope at the One limitation on such a device is the spreading of the beam due to **EXECT STATE:** The unitation on such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavelength of 1.40 nm. The element that emits l **EXECUTE AT A SET ASSET AT A SET ASSET AT A SET AND SOME INTERTATION** On such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavele **EXECT SET ASSET CONCICET SET ASSET ASSET AS A CONCICT ON SOME INTEGRED ASSET AND A laser operating at a wavelength of 1.40 nm. The element that ansis light is the end of a wire with diameter 0.200 mm.
(a) Calculate the di EXECT THE CONSTER CONSTER CONSTERED S 1111**

One limitation on such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavelength of **PROBLEMS** 1111

One limitation on such a device is the spreading of the beam due to

diffraction, with resulting dilution of beam intensity. Consider such

a laser operating at a wavelength of 1.40 nm. The element that

e tensity at the target to that at the end of the wire? (The laser is **EXECTE CONSOCTE S 1111**

One limitation on such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavelength of 1.40 nm. The eleme One limitation on such a device is the spreading of the beam due to diffraction, with resulting dilution of beam intensity. Consider such a laser operating at a wavelength of 1.40 nm. The element that emits light is the e

••30 \bullet \bullet Floaters. The floaters you see when viewing a \bullet 36-43 is a photograph in which the Moon is obscured. There are height footypedes beginning a height footypedes beginning of defects two faint, colored ring dot's distance L from your eye (or eye lens) until the dot and the
circle of the first minimum in the diffraction pattern appear to
rings) in the patterns. (Colors in other parts of the pattern overlap ••• 34 **Co** $\bullet\bullet\bullet\bullet\bullet\bullet\bullet\bullet$ A circular obstacle produces the same diffraction pattern as a circular hole of the same diameter (except very near $\theta = 0$). Airborne water drops are examples of such obstacles. When One miniation of such a welcome is the spicaling of the Centra due to a laser operating at a wavelength of 1.40 nm. The element that emits light is the end of a wire with diameter 0.200 nm.
(a) Calculate the diameter of t posite of the central diffraction maxima of those drops forms a a taser by elading a a wavelength of 1.40 time. The element that diameter 0.200 mm.
(a) Calculate the diameter of the central beam at a target 2000 km
away from the beam source. (b) What is the ratio of the beam in-
tensi Can Calculate the diameter of the central beam at a target 2000 km
away from the beam source. (b) What is the ratio of the beam in-
tensity at the target to that at the end of the wire? (The laser is
fired from space, so (a) Calculate the diameter of the Central beam at a target 2000 kin away from the beam inverse. (b) What is the ratio of the beam in-
tensity at the target to that at the end of the wire? (The laser is
fired from space, s A and y more wears somets. (b) what is the ratio of the beam in-
the strength at the barget to that at the end of the wire? (The laser is
fired from space, so neglect any atmospheric absorption.)
****34** \bullet \bullet \bullet A c **Example 19** at the canget of that at the club of the whet? (The laster is fired from space, so neglect any atmospheric absorption.)
 ••34 CD \blacktriangleright A circular bolstacle produces the same diffraction pattern as a circu somewhat larger ring is on the outer edge of the smallest of the **secondary** A circular obstacle produces the same diffraction pattern as a circular hole of the same diameter (except very near $\theta = 0$). Airborne water drops are examples of such obstacles. When you see the Moon through ble because the rings are adjacent to the diffraction minima (dark $\theta = 0$). Airborne water drops are examples of such obstacles. When
you see the Moon through suspended water drops, such as in a fog,
you intercept the diffraction pattern from many drops. The com-
posite of the central d too much to be visible.) white region that surrounds the Moon and may obscure it. Figure 36-43 is a photograph in which the Moon is obscured. There are two faint, colored rings around the Moon (the larger one may be too faint to be seen in your c 36-43 is a photograph in which the Moon is obscured. There are two faint, colored rings around the Moon (the larger one may be too faint to be seen in your copy of the photograph). The smaller tring is on the outer edge of

(a) What is the color of these rings on the outer edges of the diffraction maxima? (b) The colored ring around the central maxhave about the same diameter.Approximately what is that diameter?

Pekka Parvianen/Photo Researchers, Inc.

Figure 36-43 Problem 34.The corona around the Moon is a composite of the diffraction patterns of airborne water drops.

Module 36-4 Diffraction by a Double Slit

•35 Suppose that the central diffraction envelope of a double-slit diffraction pattern contains 11 bright fringes and the first diffrac-Pekka Parvianen/Photo Researchers, Inc.
 Figure 36-43 Problem 34. The corona around the Moon is a composite

of the diffraction patterns of airborne water drops.
 Module 36-4 Diffraction by a Double Slit
 Module 36-4 many bright fringes lie between the first and second minima of the diffraction envelope? Pekka Parvianen/Photo Researchers, Inc.
 Figure 36-43 Problem 34. The corona around the Moon is a composite

of the diffraction patterns of airborne water drops.
 Module 36-4 Diffraction by a Double Slit
 CFS Suppose

•36 A beam of light of a single wavelength is incident perpendic-

1112 CHAPTER 36 DIFFRACTION

1112 CHAPTER 36 DIFFRACTION
are each 46 μ m and the slit separation is 0.30 mm. How many the first diffraction-envelope
complete bright fringes appear between the two first-order minima maximum in a double-slit p
of the complete bright fringes appear between the two first-order minima of the diffraction pattern?

1112 CHAPTER 36 DIFFRACTION

are each 46 μ m and the slit separation is 0.30 mm. How many the first diffraction-envelope mini

complete bright fringes appear between the two first-order minima maximum in a double-slit p 1112 **CHAPTER 36 DIFFRACTION**
are each 46 μ m and the slit separation is 0.30 mm. How many the first diffraction
complete bright fringes appear between the two first-order minima maximum in a of
the diffraction pattern? central diffraction envelope?

1112 CHAPTER 36 DIFFRACTION

are each 46 μ m and the slit separation is 0.30 mm. How many

the first diffraction-

complete bright fringes appear between the two first-order minima

and $a = 30.0 \mu$ m? (b
 •37 In a doub within the second side peak of the diffraction envelope and diffrac-1112 CHAPTER 36 DIFFRACTION

are each 46 μ m and the slit separation is 0.30 mm. How many the first diffraction-envelope m

complete bright fringes appear between the two first-order minima

of the diffraction pattern?
 the ratio of the slit separation to the slit width?

are each 46 μ m and the slit separation is 0.30 mm. How many

the fincomplete bright fringes appear between the two first-order minima

of the diffraction pattern?
 ••37 In a double-slit experiment, the slit separatio yielding a diffraction pattern whose graph of intensity I versus and about 26° in light with a wavelength of 550 nm. What is the grating gular position θ is shown in Fig. 36-44. Calculate (a) the slit width spacing of complete bright fringes appear between the two first-order minima

of the diffraction pattern?
 S7 In a double-slit experiment, the slit separation d is 2.00 times

the slit width w. How many bright interference fringes **and** $a = 3$
 **and intensity of the slit separation. All the slit separation envelope?

agging a** diffraction envelope and diffraction
 all the secon the $m = 1$ and $m = 2$ interference fringes.

Figure 36-44 Problem 39.

••40 **GO** Figure 36-45 gives the pasine of the angle θ in a two-slit interference experiment using light of scale is set by $\beta_s = 80.0$ rad. What are ¹

(a) the slit separation, (b) the smallest angle for a maxima, and (d) the sparater β of Eq. 36-20 versus the parameter β of Eq. 36-20 versus the sine of the angle θ in a two-slit inter-

ference experiment u number of interference maxima (count them on both sides of the **Figure 36-44 Problem 39.**
 Exampleme 36-45 gives the pattern integrating Two on a diffraction grating Two on a diffraction grading Two one and $\theta = 0.2$ and sin $\theta = 0.3$ ranneter β of Eq. 36-20 versus the β_i (a **Example 19** or a minimum of the two-slit interference experiment of Fig. 35-10, the slit separation between the angle *θ* in a two-slit interference experiment using light of the collargest, wavelength 435 nm. The verti

greatest angle for a minimum? Assume that none of the interference maxima are completely eliminated by a diffraction minimum.

widths are each 12.0 μ m, their separation is 24.0 μ m, the wavelength is grating. (a) What is the angular separation between the second-Example of the solution of the solution of the set at which use and the collargest of dy second largest, (d) second the screen is set by $\beta_s = 80.0$ rad. What are a specifier of interference maxima is $\frac{1}{2}$ or $\frac{1}{2$ sentence experiment difference experiment of Fig. 35-10, the sit at wide at which intensity at the maximum on which it lies of the maxima are completely eliminated by a diffraction minimum.
 Example the interference max What is the ratio of I_P to the intensity I_m at the center of the pattern? (c) What is the highest order for which maxima for both wave-(b) Determine where P is in the two-slit interference pattern by giving the maximum or minimum on which it lies or the maximum and minimum between which it lies. (c) In the same way, for the diffraction that causes diffraction to eliminate the fourth bright scales of the cause of the maniform on both sides angle for a maxima, and (d) the set fits these d occurs, determine where point P is in the diffraction pattern. **Example 19** for a minimum? Assume that none of the interference
 Example 19 of a minimum? Assume that none of the interference
 Example 19 Constant and a let two-slit interference experiment of Fig. 35-10, the slit
 (a) In the two-slit interference experiment of Fig. 35-10, the slit in and $\lambda_2 = 500$ nm with a light sign widths are each 12.0 μ m, their separation is 24.0 μ m, the wavelength is 600 nm, and the viewing screen

causes diffraction to eliminate the fourth bright side fringe? which the complete wavelength range of the beam is present? In (b) What other bright fringes are also eliminated? (c) How many that highest order, at what angle does the light at wavelength (c) other ratios of d to a cause the diffraction to (exactly) eliminate 460.0 nm and (d) 640 that bright fringe?

•• 43 SSM WWW (a) How many bright fringes appear between

the first diffraction-envelope minima to either side of the central
maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm,
and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to t maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm, ither side of the central
550 nm, $d = 0.150$ mm,
he intensity of the third
ringe? the central
0.150 mm,
of the third and $a = 30.0 \ \mu m$? (b) What is the ratio of the intensity of the third bright fringe to the intensity of the central fringe? the first diffraction-envelope minima to either side of the central
maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm,
and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to t

Module 36-5 Diffraction Gratings

beetles (whirligig beetles) are colored by optical interference that is due to scales whose alignment forms a diffraction grating (which the first diffraction-envelope minima to either side of the central
maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm,
and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to t the first diffraction-envelope minima to either side of the central
maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm,
and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to t order maxima (on opposite sides of the zeroth-order maximum) is the first diffraction-envelope minima to either side of the central
maximum in a double-slit pattern if $\lambda = 550$ nm, $d = 0.150$ mm,
and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to t spacing of the beetle? and $a = 30.0 \mu m$? (b) What is the ratio of the intensity of the third
bright fringe to the intensity of the central fringe?
Module 36-5 Diffraction Gratings
44 \blacktriangleright Perhaps to confuse a predator, some tropical gyri bright fringe to the intensity of the central fringe?
 Module 36-5 Diffraction Gratings
 A4 \bullet Perhaps to confuse a predator, some tropical gyrinid

beetles (whirligig beetles) are colored by optical interference **Module 36-5 Diffraction Gratings**
 44 $\overline{ }$ Perhaps to confuse a predator, some tropical gyrinid

beetles (whirligig beetles) are colored by optical interference that

is due to scales whose alignment forms a diffrac rays are perpendicular to the grating, the angle between the first-
order maxima (on opposite sides of the zeroth-order maximum) is
about 26° in light with a wavelength of 550 nm. What is the grating
spacing of the beetle

values of θ at which maxima appear on a distant viewing screen? order maxima (on opposite sides of the zeroth-order maximum) is
about 26° in light with a wavelength of 550 nm. What is the grating
spacing of the beetle?
45 A diffraction grating 20.0 mm wide has 6000 rulings. Light of about 26° in light with a wavelength of 550 nm. What is the grating
spacing of the beetle?
45 A diffraction grating 20.0 mm wide has 6000 rulings. Light of
wavelength 589 nm is incident perpendicularly on the grating.
W

•46 Visible light is incident perpendicularly on a grating with 315 rulings/mm.What is the longest wavelength that can be seen in the fifth-order diffraction?

fraction experiment, in addition to the $m = 0$ order? **47 SSM** ILW A grating has 400 lines/mm. How many orders of

spacing of the beetle?
 •45 A diffraction grating 20.0 mm wide has 6000 rulings. Light of wavelength 589 nm is incident perpendicularly on the grating.

What are the (a) largest, (b) second largest, and (c) third larges **45** A diffraction grating 20.0 mm wide has 6000 rulings. Light of wavelength 589 nm is incident perpendicularly on the grating. What are the (a) largest, (b) second largest, and (c) third largest values of θ at which plane waves of wavelength $\lambda = 600$ nm at normal incidence. rependicularly on the grating.

d largest, and (c) third largest

on a distant viewing screen?

dicularly on a grating with 315

relength that can be seen in the

lines/mm. How many orders of

00 nm) can it produce in a d What are the (a) largest, (b) second largest, and (c) third largest
values of θ at which maxima appear on a distant viewing screen?
 46 Visible light is incident perpendicularly on a grating with 315

rulings/mm. Wha values of θ at which maxima appear on a distant viewing screen?
 46 Visible light is incident perpendicularly on a grating with 315

rulings/mm. What is the longest wavelength that can be seen in the

fifth-order dif order if the grating has 1000 slits? rulings/mm. What is the longest wavelength that can be seen in the fifth-order diffraction?
 •47 SSM LIW A grating has 400 lines/mm. How many orders of the entire visible spectrum (400–700 nm) can it produce in a dif-
 Example 1.1 Conducts and the symbol sum and the symbol sum and the symbol spectrum (400–700 nm) can it produce in a diffraction experiment, in addition to the $m = 0$ order?
 Conducts and the symbol spectrum (400–700

by sin $\theta = 0.2$ and sin $\theta = 0.3$. The fourth-order g has 400 lines/mm. How many orders of
m (400–700 nm) can it produce in a dif-
dition to the $m = 0$ order?
g is made up of slits of width 300 nm with
rating is illuminated by monochromatic
ugth $\lambda = 600$ nm at normal inci the entire visible spectrum (400–700 nm) can it produce in a dif-
fraction experiment, in addition to the $m = 0$ order?
••48 A diffraction grating is made up of slits of width 300 nm with
separation 900 nm. The grating **Fraction experiment, in addition to the** $m = 0$ **order?**
 ••48 A diffraction grating is made up of slits of width 300 nm with separation 900 nm. The grating is illuminated by monochromatic plane waves of wavelength $\lambda =$ **48** A diffraction grating is made up of slits of width 300 nm with
separation 900 nm. The grating is illuminated by monochromatic
plane waves of wavelength $\lambda = 600$ nm at normal incidence.
(a) How many maxima are there order number m of the maxima produced by the grating? (a) How many maxima are there in the full diffraction pattern?

(b) What is the angular width of a spectral line observed in the first

order if the grating has 1000 slits?
 •49 SSM WWW Light of wavelength 600 nm is inc and the first

e observed in the first

m is incident normally

occur at angles given

r maxima are missing.

slits? (b) What is the

nat slit width, what are

d largest values of the

e grating?

ube incident normally

a 67.18°, or a singular manufacture of the grating has 1000 slits?
 •49. SSM. WWW. Light of wavelength 600 nm is incident normally

on a diffraction grating. Two adjacent maxima occur at angles given

by sin $\theta = 0.2$ a in since

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in the state of the

in sing.

in that

on m

on the

second-

imallest •49 SSM WWW Light of wavelength 600 nm is incident normally (a) How many maxima are there

(b) What is the angular width of a s

order if the grating has 1000 slits?
 age of the grating has 1000 slits?
 age on a diffraction grating. Two adjace

on a diffraction grating. Two ad

 $\sin \theta$ ••50 With light from a gaseous discharge tube incident normally light are experimentally found at angles $\theta = \pm 17.6^{\circ}, 37.3^{\circ}, -37.1^{\circ}$, best fits these data.

••51 A diffraction grating having 180 lines/mm is illuminated with a light signal containing only two wavelengths, $\lambda_1 = 400$ nm and λ_2 = 500 nm. The signal is incident perpendicularly on the ⁵⁰ signal is the separation between adjacent slits? (b) What is the it width this grating can have? For that slit width, what are gest, (d) second largest, and (e) third largest values of the ber *m* of the maxima produ (a) What is the angular separation between the second-
order manifest slit width this grating can have? For that slit width, what are
the (c) largest, (d) second largest, and (e) third largest values of the
order number order maxima of these two wavelengths? (b) What is the smallest angle at which two of the resulting maxima are superimposed? **•50** With light from a gaseous discharge tube incident normally
on a grating with slit separation 1.73 μ m, sharp maxima of green
light are experimentally found at angles $\theta = \pm 17.6^{\circ}$, 37.3°, -37.1° ,
65.2°, an lengths are present in the diffraction pattern? 65.2°, and -65.0°. Compute the wavelength of the green light that
best fits these data.
 •51 C A diffraction grating having 180 lines/mm is illuminated

with a light signal containing only two wavelengths, $\lambda_1 = 400$ **Example 160** and the state of the lines/mm is illuminated with a light signal containing only two wavelengths, $\lambda_1 = 400$ nm and $\lambda_2 = 500$ nm. The signal is incident perpendicularly on the grating. (a) What is the ang the (c) targest, (d) second largest, and
order number m of the maxima product
 $\frac{\sin \theta}{\sin \theta}$
1 order number m of the maxima product
on a grating with slit separation 1.73
bblem 40. light are experimentally found at ang

•• 52 ^{co} A beam of light consisting of wavelengths from overlapped by another order? (b) What is the highest order for **Example 18** A diffurted parallelocal and containing only two wavelengths, $\lambda_1 = 400$ nm and $\lambda_2 = 500$ nm. The signal is incident perpendicularly on the grating. (a) What is the angular separation between the second-
o with a fight signal containing omly two wavelengths, $A_1 = 400$ film
grating. (a) 00 nm. The signal is incident perpendicularly on the
grating. (a) What is the angular separation between the second-
order maxima of these and $x_2 = 300$ film. The signar is incluent perpendicularly on the grading. (a) What is the angular separation between the second-
order maxima of these two wavelengths? (b) What is the smallest
angle at which two of the grating. (a) what is the angular separation between the second-
order maxima of these two wavelengths? (b) What is the smallest
angle at which two of the resulting maxima are superrimposed?
(c) What is the highest order f

•• 53 \bullet A grating has 350 rulings/mm and is illuminated at normal

incidence by white light. A spectrum is formed on a screen 30.0 cm
from the grating. If a hole 10.0 mm square is cut in the screen its
incer edge being 50.0 mm from the central maximum and parallel 39.8 pm. If the reflecti incidence by white light. A spectrum is formed on a screen 30.0 cm

from the grating. If a hole 10.0 mm square is cut in the screen, its

from the grating. Supplementary and parallel

inter edge being 50.0 mm from the cen incidence by white light. A spectrum is formed on a screen 30.0 cm
from the grating. If a hole 10.0 mm square is cut in the screen, its
inner edge being 50.0 mm from the central maximum and parallel 39.8 pm
to it, what ar incidence by white light. A spectrum is formed on a screen 30.0 cm
from the grating. If a hole 10.0 mm square is cut in the screen, its
inner edge being 50.0 mm from the central maximum and parallel 39.8 pm. If the r
to i light that passes through the hole?

"grating":

$$
I = \frac{1}{9}I_m(1 + 4\cos\phi + 4\cos^2\phi),
$$

where $\phi = (2\pi d \sin \theta)/\lambda$ and $a \ll \lambda$.

Module 36-6 Gratings: Dispersion and Resolving Power

•55 SSM ILW A source containing a mixture of hydrogen and From the grating. If a hole 10.0 mm square is cut in the screen, its

from the grating. If a hole 10.0 mm square is cut in the screen, its

tight that passes through the hole?
 tight that are the (a) shortest and (b) lo Find the state is set by $\theta_s = 2.00^\circ$. The better in the separation is 0.180 nm. Find the minimum
these lines in the first order. The minimum and parallel 39.8 pm. If the reflection from
to it, what are the (a) shortest number of lines needed in a diffraction grating that can resolve these lines in the first order. "grating":

"grating": $I = \frac{1}{9}I_m(1 + 4 \cos \phi + 4 \cos^2 \phi)$, scale is set by $\theta_3 = 2.00^\circ$.

where $\phi = (2\pi d \sin \theta)/\lambda$ and $a \ll \lambda$.

(a) shorter and (b) longers and **Mechanism** and **Mechanism** and **Mechanism** (a) shorter and (where $\phi = (2\pi d \sin \theta)/\lambda$ and $a \ll \lambda$.
 Module 36-6 Gratings: Dispersion and Resolving Power
 •55 SSM LIW A source containing a mixture of hydrogen and

deuterium atoms emits red light at two wavelengths whose mean is

•56 (a) How many rulings must a 4.00-cm-wide diffraction grating have to resolve the wavelengths 415.496 and 415.487 nm in the second have to resolve the wavelengths 413.490 and 413.467 film in the second
order? (b) At what angle are the second-order maxima found?

ond-order (c) D and (d) R , and the third-order (e) D and (f) R ? ber of lines needed in a diffraction grating that can resolve

elines in the first order.

(a) How many rulings must a 4.00-cm-wide diffraction grating

to resolve the wavelengths 415.496 and 415.487 nm in the second

7? **•58** (a) How many rulings must a video diffraction grating
 •57 Light at wavelengths 415.496 and 415.487 nm in the second

order? (b) At what angle are the second-order maxima found?
 •57 Light at wavelength 589 nm f **100** (a) How many rulings must a 4.00-cm-wide diffraction grating
have to resolve the wavelengths 415.496 and 415.487 nm in the second
order? (b) At what angle are the second-order maxima found?
1000
1000 1000 100 have to resolve the wavelengths 415.496 and 415.487 nm in the second
order? (b) At what angle are the second-order maxima found?
 F57 Light at wavelength 589 nm from a sodium lamp is incident per-

are the first-order (

the smallest wavelength interval it can resolve in the third order at \bullet 69 X rays of wavelength 0.12 nm are found to undergo second-
 $\lambda = 500 \text{ nm}^2$ (b) How many higher orders of maxima can be seen? order reflection at λ = 500 nm? (b) How many higher orders of maxima can be seen?

resolve in the second order?

 \bullet 60 The *D* line in the spectrum of sodium is a doublet with wavependicularly on a grating with 40 000 rulings over width 76 nm. What
are the first-order (a) dispersion D and (b) resolving power R, the sec-
ond-order (c) D and (d) R, and the third-order (e) D and (f) R?
 ***58** A gratin of lines needed in a grating that will resolve this doublet in the crystal. What is the unit cell size a_0 ? second-order spectrum.

•61 With a particular grating the sodium doublet $(589.00 \text{ nm and}$ of x rays of wavelength 0.125 nm be the smallest wavelength interval it can resolve in the third order at $\frac{1}{\sqrt{5}}$ order reflection at order the second price of maxima can be seen?
 59 A diffraction grating with a width of 2.0 cm contains the crystal. $\lambda = 500 \text{ nm}$? (b) How many higher orders of maxima can be seen? correlation at example that width. For an incident wavelength of the exystal?

1000 lines/cm across that width. For an incident wavelength of the exystal?
 of the rulings.

••62 A diffraction grating illuminated by monochromatic light 600 nm, what is the smallest wavelength difference this grating can

resolve in the second order?
 CO The D line in the spectrum of sodium is a doublet with wave-

lengths 589.0 and 589.6 mm. Calculate the minimum numbe the product of that line's half-width and the grating's resolving power? (b) Evaluate that product for the first order of a grating of lengths 589.0 and 589.6 nm. Calculate the minimum number

of lines needed in a grating that will resolve this doublet in the

second-order spectrum.
 •61 With a particular grating the sodium doublet (589.00 nm and
 •61 second-order spectrum.
 •61 With a particular grating the sodium doublet (589.00 nm and

589.59 nm) is viewed in the third order at 10° to the normal and is

barely resolved. Find (a) the grating spacing and (b) the tot **302** A diffraction grating illuminated by monochromatic light

the product of that line's half-width and the grating's resolving

turned through angle ϕ is

power? (b) Evaluate that product for the first order of a gr

••63 Assume that the limits of the visible spectrum are arbitrarily give diffraction maxima. What are absence at $\frac{20}{2}$ can $\frac{20}{2}$ can Galaxima Changer with the (a) smaller and (b) larger value limeter of a grating that will spread the first-order spectrum through an angle of 20.0°.

Module 36-7 X-Ray Diffraction

calcite crystal?

•65 An x-ray beam of wavelength A undergoes first-order reflection ated order number m and the (c) shortest λ and (d) associated m of (Bragg law diffraction) from a crystal when its angle of incidence to a **Example 1 CRICITY Example 1 CRICITY Example 1 CRICITY Example 1 CRICITY EXECTS** Assume that the limits of the visible spectrum are arbitrarily the (a) smaller and (b) la limeter of a grating that will spr Assuming that the two beams reflect from the same family of reflecting

•66 An x-ray beam of a certain wavelength is incident on an NaCl crystal, at 30.0° to a certain family of reflecting planes of spacing **EXECREMS** 1113
 CRECREMS 1113
 CRECREMS An x-ray beam of a certain wavelength is incident on an NaCl

crystal, at 30.0° to a certain family of reflecting planes of spacing

39.8 pm. If the reflection from those planes **39.8** PROBLEMS **1113**
 39.8 pm. If the reflection from those planes is of the first order,
 39.8 pm. If the reflection from those planes is of the first order,
 39.8 pm. If the reflection from those planes is of th what is the wavelength of the x rays?

••54 Derive this expression for the intensity pattern for a three-slit for the diffraction of an x-ray beam by a crystal. The horizontal •67 Figure 36-46 is a graph of intensity versus angular position θ **FROBLEMS** 1113
 FROBLEMS 1113
 FROBLEMS 1113
 FROBLEMS 1113
 CONTITE CONTABY CONTING THE DETECT AND ANDED THE HORIZON OF A SUMPTION OF A SUMPTION of the first order,

what is the wavelength of the x rays?
 FROBL scale is set by $\theta_s = 2.00^\circ$. The beam consists of two wavelengths, and the spacing between the reflecting planes is 0.94 nm. What are the **PROBLEMS** 1113

2.00°. The beam consider a certain family of reflecting planes of spacing

flection from those planes is of the first order,

1.00° mgth of the x rays?

is a graph of intensity versus angular position θ **EXECT 1113**
 EXECT ALC
 (a) shorter and (b) longer wavelengths in the beam?

the crystal? **••**
 •• θ (degrees)
 •• θ (degrees)
 •• •• 17 Figure 36-46 Problem 67.
 ••68 If first-order reflection occurs in a crystal at Bragg angle 3.4°, at what Bragg angle does second-order reflection occur f **EXECUTE:**
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 EXECUTE:
 ESS If first-order reflection occurs in a crystal at Bragg angle is
 ESS X rays of wavelength 0.12 nm are found to undergo seconder reflection at If first-order reflection occurs in a crystal at Bragg an

that Bragg angle does second-order reflection occur f

e family of reflecting planes?

X rays of wavelength 0.12 nm are found to undergo

or reflection at a Bragg at what Bragg angle does second-order reflection occur
same family of reflecting planes?
 CO X rays of wavelength 0.12 nm are found to undergo

order reflection at a Bragg angle of 28° from a lithium

crystal. What is t

flection from the reflection planes shown occurs when an x-ray beam of $\theta = 63.8^{\circ}$ with the top face of the

incident on an NaCl crystal at angle crystal. What is the interplanar spacing of the reflectin
the crystal?
 10 CD In Fig. 36-47, first-order re-

flection from the reflection planes

shown occurs when an x-ray beam of

wavelength 0.260 nm makes an angle
 tal and a family of reflecting planes. Let the reflecting planes have separation $d = 0.252$ nm. The crystal is turned through angle ϕ around an axis perpendicular to the plane of the beam page until these reflecting planes $\theta = 0.5$. What is the unit cell size a_0 ?
 •71 WWW In Fig. 36-48, let a beam

of x rays of wavelength 0.125 nm be

incident on an NaCl crystal at angle
 $\theta = 45.0^\circ$ to the top face of the crys-

ration d = 0.252 nm. crystal. What is the unit cell size a_0 :
 •71 WWW In Fig. 36-48, let a beam

of x rays of wavelength 0.125 nm be

incident on an NaCl crystal at angle
 $\theta = 45.0^{\circ}$ to the top face of the crys-

figure 36-47 Problem of ϕ if the crystal is turned clockwise and the (c) smaller and (d) larger value of ϕ if it is turned counterclockwise? Let the reflecting planes.

Let the reflecting planes have separation $d = 0.252$ nm. The crystal is

turned through angle ϕ around an

page until these reflecting planes

give diffraction maxima. What are

the (a) smal $u = 0.252$ init. The crystal is
through angle ϕ around an
mtil these reflecting planes
ffraction maxima. What are
smaller and (b) larger value
the crystal is turned clockwise
 ϕ if it is turned counter-
 ϕ if it is turned through angle φ around an Incident

page until these reflecting planes

give diffraction maxima. What are

the (a) smaller and (b) larger value

and the (c) smaller and (d) larger

and the (c) smaller and (d) l $\theta = 45.0^{\circ}$ to the top face of the crys-**WWW** In Fig. 36-48, let a beam
ays of wavelength 0.125 nm be
nt on an NaCl crystal at angle
 5.0° to the top face of the crys-
Figure 36-47 I

•64 What is the smallest Bragg angle for x rays of wavelength •• 72 In Fig. 36-48, an x-ray beam of wavelengths from 95.0 to 140 pm is incident at $\theta = 45.0^{\circ}$ to a family of reflecting planes with spacing $d = 275$ pm. What are the (a) longest wavelength λ and (b) associgive diffraction maxima. What are

the (a) smaller and (b) larger value

of ϕ if the crystal is turned clockwise

and the (c) smaller and (d) larger
 ϕ if it is turned counter-
 ϕ if it is turned counter-
 ϕ the (a) smaller and (b) larger value

of ϕ if the crystal is turned clockwise

and the (c) smaller and (d) larger

Figure 36-48 Problems 71

value of ϕ if it is turned counter-

clockwise?
 -72 In Fig. 36-48, an x and the (c) smaller and (d) larger **Figure 36-48** Problems 71 value of ϕ if it is turned counter-
clockwise?
***72** In Fig. 36-48, an x-ray beam of wavelengths from 95.0 to 140 pm is incident at $\theta = 45.0^{\circ}$ to a fami

the intensity maxima in the diffraction of the beam?

third-order reflection when its angle of incidence to that face is 60°. one side of the structure shown in Fig. 36-28a. The largest interplanar planes, find (a) the interplanar spacing and (b) the wavelength A. sketch the (a) second largest, (b) third largest, (c) fourth largest, (d) spacing of reflecting planes is the unit cell size a_0 . Calculate and Froblems 71

and 72.

froblems 71

and 72.

from 95.0 to 140

lanes with spaci-

and (b) associated *m* of

ructure, such as

gest interplanar

Calculate and

urth largest, (d)

1114 CHAPTER 36 DIFFRACTION
fifth largest, and (e) sixth largest interplanar spacing. (f) Show that Show that bright fringes occ
your results in (a) through (e) are consistent with the general formula
 $d = \frac{a_0}{\sqrt{(\text{compare$ your results in (a) through (e) are consistent with the general formula fifth largest, and (e) sixth largest interplanar spacing. (f) Show that Show that bright fringes occur at angles θ that satisfy the equation your results in (a) through (e) are consistent with the general formula $d(\sin \$

$$
d = \frac{a_0}{\sqrt{h^2 + k^2}},\tag{6}
$$

factor other than unity).

Additional Problems

74 An astronaut in a space shuttle claims she can just barely refifth largest, and (e) sixth largest interplanar spacing. (f) Show that Show that bright fringes occur
your results in (a) through (e) are consistent with the general formula
 $d = \frac{a_0}{\sqrt{h^2 + k^2}}$,

where h and k are re fifth largest, and (e) sixth largest interplanar spacing. (f) Show that

your results in (a) through (e) are consistent with the general formula
 $d(\sin \psi + \sin \theta) = m\lambda$
 $d(\sin \psi + \sin \theta) = m\lambda$

(Compare this equation with
 $\psi = 0$ ideal conditions. Take $\lambda = 540$ nm and the pupil diameter of the asargest interplanar spacing. (f) Show that

e) are consistent with the general formula
 $\sqrt{h^2 + k^2}$, (Co.

y prime integers (they have no common

find

the pupil diameter of the as-

is on Earth's surface, 160 km below.
 your results in (a) through (e) are consistent with the general formula
 $d = \frac{a_0}{\sqrt{h^2 + k^2}}$,

where h and k are relatively prime integers (they have no common

factor other than unity).
 Additional Problems
 74 An where h and k are relatively prime integers (they have no common
 $\sqrt{h^2 + k^2}$, $\psi = 0$ has been treated in this

factor other than unity).
 Additional Problems
 Cator other than unity).
 Additional Problems
 Cato where *h* and *k* are relatively prime integers (they have no common
factor other than unity).
Additional Problems
Additional Problems
Additional Problems
Additional Problems
Additional Problems
Additional Prob Aditional Problems
 Aditional Problems
 Aditional Problems
 A a astronaut in a space shuttle claims she can just barely re-
 A and solve two point sources on Earth's surface, 160 km below.

Calculate their (a) a

75 SSM Visible light is incident perpendicularly on a diffraction separation to slit w with an intensity maximum at $\theta = 30.0^{\circ}$?

Additional Problems
 Calculate their (a) angular and (b) linear separation, assuming coder maximum from the incident c

Solve two point sources on Earth's surface, 160 km below.

Calculate their (a) angular and (b) li **74** An astronaut in a space shuttle claims she can just barely re-

solve two point sources on Earth's surface, 160 km below.

Calculate their (a) angular and (b) linear separation, assuming

in the central peak of the d number of lines required for the two wavelengths to be resolved in the second order? deal conditions. Take $\lambda = 540$ nm and the pupil diameter of the as-
 75 SSM Visible light is incident perpendicularly on a diffraction experiment of the first s

dop rulings/mm. What are the (a) longest, (b) second

lo **EXECUTE:** THE INTERT IN the spectral of the spectral of the spectral of the light is incident perpendicularly on a diffraction separation to slit grating of 200 rulings/mm. What are the (a) longest, (b) second tral diffr Exerce the (a) longest, (b) second

tradiffraction

engths that can be associated
 0.0° ?

b wavelengths, 590.159 nm and
 450 nm is incitive a diffraction grating. If the

wo-slit interfere

with a diffraction gra grating of 200 rulings/mm. What are the (a) longest, (b) second
longest, and (c) third longest wavelengths that can be associated
with an intensity maximum at $\theta = 30.0^{\circ}$?
76 A beam of light consists of two wavelengt longest, and (c) third longest wavelengths that can be associated
 78 A beam of light with a narrow
 85 A bea with an intensity maximum at $\theta = 30.0^{\circ}$?
 26 A beam of light consists of two wavelengths, 590.159 nm and

590.220 nm, that are to be resolved with a diffraction grating. If the

grating has lines across a width of 3

intensity for blue-green light ($\lambda = 500$ nm) at the same angle of mum of intensity for orange light ($\lambda = 600$ nm) and a minimum of 77 SSM In a single-slit diffraction experiment, there is a mini-

76 A beam of light consists of two wavelengths, 590.159 nm and 450 nm is incident per solution and a slime same of lines are costed with a diffraction grating. If the width of 1.80 cm and grating has lines across a widt number of complete bright fringes appearing between the two first-order minima of the diffraction pattern? (Do not count the has a single wavelength of 550 nm and that your pupil has a diamfringes that coincide with the minima of the diffraction pattern.) eter of 5.5 mm. **77 SSM** In a single-slit diffraction experiment, there is a mini-
mum of intensity for orange light ($\lambda = 600$ nm) and a minimum of
intensity for blue-green light ($\lambda = 500$ nm) at the same angle of
has a diameter of 4.0

79 SSM A diffraction grating has resolving power $R = \lambda_{\text{avg}}/\Delta \lambda = 88$ In a sing just be resolved is given by $\Delta f = c/Nm\lambda$. (b) From Fig. 36-22, show that the times required for light to travel along the ray at the bottom of the figure and the ray at the top differ by $\Delta t = (N d/c) \sin \theta$. 78 **a** A double-slit system with individual slit widths of 0.030 mm
and a slit separation of 0.18 mm is illuminated with 500 nm light di-
pendicular to you
nected perpendicular to the plane of the slits. What is the total m with individual slit widths of 0.030 mm

8 mm is illuminated with 500 nm light di-

bendicular to yche

the plane of the slits. What is the total

from the flowers

the flowers the ringes appearing between the two

ing rected perpendicular to the plane of the slits. What is the total

from the flowers when they are at t

number of complete bright fringes appearing between the two

first-order minim of the diffraction pattern? (Do not co The publishing a pearing between the two ing to the Rayleight first-order minima of the diffraction pattern? (Do not count the has a single waveler thrigges that coincide with the minima of the diffraction pattern.) **79** fringes that coincide with the minima of the diffraction pattern.)
 are all the SEM A diffraction grating has resolving power $R = \lambda_{\text{avg}}/\Delta \lambda = 88$ In a
 Nm. (a) Show that the corresponding frequency range Δf that **79 SSM** A diffraction grating has resolving power $R = \lambda_{avg}/\Delta$
 Nm. (a) Show that the corresponding frequency range Δf that is just be resolved is given by $\Delta f = c/Nm\lambda$. (b) From Fig. 36-22, sh that the times required Nm. (a) Show that the corresponding frequency range Δf that can

that the sit width to the wavelength

that the times required for light to travel and grad the ray at the top differ by $\Delta t = c(Nd/c)$ sin θ .

that the f

According to Rayleigh's criterion, what distance apart must two
small objects be if their images are just barely resolved when they
 $5.10 \mu m$. The viewing screen is 3.20 m distant. On the screen, what

$$
d(\sin \psi + \sin \theta) = m\lambda, \text{ for } m = 0, 1, 2, \dots
$$

r at angles θ that satisfy the equation $m\lambda$, for $m = 0, 1, 2, ...$
ith Eq. 36-25.) Only the special case is chapter. t satisfy the equation
0, 1, 2,
0 Only the special case Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82 $\psi = 0$ has been treated in this chapter.

where h and k are relatively prime integers (they have no common
incidence by light of wavelength 600 nm. Plot, as a function of the
fector other than unity) 82 A grating with $d = 1.50 \mu m$ is illuminated at various angles of Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82 Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82 Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82

Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82 Show that bright fringes occur at angles θ that satisfy the equation
 $d(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$

(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
 82 in the central peak of the diffraction envelope and (b) how many are in either of the first side peak of the diffraction envelope? 83 SSM In two-slit interference, if the slit separation is 14 μ m and

 $a(\sin \psi + \sin \theta) = m\lambda$, for $m = 0, 1, 2, ...$
(Compare this equation with Eq. 36-25.) Only the special case
 $\psi = 0$ has been treated in this chapter.
82 A grating with $d = 1.50 \mu m$ is illuminated at various angles of
incidence separation to slit width if there are 17 bright fringes within the central diffraction envelope and the diffraction minima coincide with two-slit interference maxima?

85 A beam of light with a narrow wavelength range centered on angle of incidence (0 to 90°), the angular deviation of the first-
order maximum from the incident direction. (See Problem 81.)

83 SSM In two-slit interference, if the slit separation is 14 μ m and

the slit widths are order maximum from the incident direction. (See Problem 81.)

83 SSM In two-slit interference, if the slit separation is 14 μ m and

the slit widths are each 2.0 μ m, (a) how many two-slit maxima are

in the central p **83 SSM** In two-slit interference, if the slit separation is 14 μ m and
the slit widths are each 2.0 μ m, (a) how many two-slit maxima are
in the central peak of the diffraction envelope and (b) how many
are in either grating can resolve in the third order? in the central peak of the diffraction envelope and (b) how many
are in either of the first side peak of the diffraction envelope?
84 C In a two-slit interference pattern, what is the ratio of slit
separation to slit wi separation to slit width if there are 17 bright fringes within the central diffraction envelope and the diffraction minima coincide with
two-slit interference maxima?
85 A beam of light with a narrow wavelength range ce two-slit interference maxima?
 85 A beam of light with a narrow wavelength range centered on

450 nm is incident perpendicularly on a diffraction grating with a

width of 1.80 cm and a line density of 1400 lines/cm acro

 according to Rayleigh's criterion? Assume the pupil of your eye length (perpendicular to your line of sight) that you can resolve, the light reaching you.

($\lambda = 600$ nm) and a minimum of according to Rayleigh's criterion?
= 500 nm) at the same angle of has a diameter of 4.00 mm, and us
width is this possible? the light reaching you.
individual slit widths of 0.030 mm **87** T **85** A beam of light with a narrow wavelength range centered on 450 nm is incident perpendicularly on a diffraction grating with a width of 1.80 cm and a line density of 1400 lines/cm across that width. For this light, wh from the flowers when they are at the limit of resolution according to the Rayleigh criterion? Assume the light from the flowers width. For this light, what is the smallest wavelength difference this
grating can resolve in the third order?
86 If you look at something 40 m from you, what is the smallest
length (perpendicular to your line of sight) grating can resolve in the third order?
 86 If you look at something 40 m from you, what is thength (perpendicular to your line of sight) that you c

according to Rayleigh's criterion? Assume the pupil c

has a diameter **86** If you look at something 40 m from you, what is the smallest length (perpendicular to your line of sight) that you can resolve, according to Rayleigh's criterion? Assume the pupil of your eye has a diameter of 4.00 m according to Rayleigh's criterion? Assume the pupil of your eye
has a diameter of 4.00 mm, and use 500 nm as the wavelength of
the light reaching you.
87 Two yellow flowers are separated by 60 cm along a line per-
pendi **87** Two yellow flowers are separated by 60 cm along a line per-
pendicular to your line of sight to the flowers. How far are you
from the flowers when they are at the limit of resolution accord-
ing to the Rayleigh crite 87 Two yellow flowers are separated by 60 cm along a line per-
pendicular to your line of sight to the flowers. How far are you
from the flowers when they are at the limit of resolution accord-
ing to the Rayleigh criteri

 $\lambda_{\text{avg}}/\Delta \lambda =$ **88** In a single-slit diffraction Δf that can of the slit width to the wavelength if the second diffraction minima pattern on a viewing screen?

(c) show that $(\Delta f)(\Delta t) = 1$, this relation being independent of the action of the state of the continent of the section of the continent of lines on the grating?

A single-slit diffraction experiment is set up with light of ing to the Rayleigh criterion? Assume the light from the flowers
has a single wavelength of 550 nm and that your pupil has a diam-
eter of 5.5 mm.
88 In a single-slit diffraction experiment, what must be the ratio
of th has a single wavelength of 550 nm and that your pupil has a diam-
eter of 5.5 mm.
88 In a single-slit diffraction experiment, what must be the ratio
of the slit width to the wavelength if the second diffraction minima
a is the distance between the center of the diffraction pattern and the second diffraction minimum? of the slit width to the wavelength it the second diffraction minima
are to occur at an angle of 37.0° from the center of the diffraction
pattern on a viewing screen?
89 A diffraction grating 3.00 cm wide produces the s are to occur at an angle of 37.0° from the center of the diffraction
pattern on a viewing screen?
 89 A diffraction grating 3.00 cm wide produces the second order

at 33.0° with light of wavelength 600 nm. What is the t

ima) lie to one side of the central maximum?

92 In an experiment to monitor the Moon's surface with a light at 33.0° with light of wavelength 600 nm. What is the total number
of lines on the grating?
90 A single-slit diffraction experiment is set up with light of
wavelength 420 nm, incident perpendicularly on a slit of width
 beam, pulsed radiation from a ruby laser ($\lambda = 0.69 \ \mu m$) was directed to the Moon through a reflecting telescope with a mirror ra-90 A single-slit diffraction experiment is set up with light of wavelength 420 nm, incident perpendicularly on a slit of width 5.10 $μ$ m. The viewing screen is 3.20 m distant. On the screen, what is the distance between wavelength 420 nm, incident perpendicularly on a slit of width 5.10 μ m. The viewing screen is 3.20 m distant. On the screen, what is the distance between the center of the diffraction pattern and the second diffraction 5.10 μ m. The viewing screen is 3.20 m distant. On the screen, what
is the distance between the center of the diffraction pattern and
the second diffraction minimum?
91 A diffraction grating has 8900 slits across 1.20 is the distance between the center of the diffraction pattern and
the second diffraction minimum?

91 A diffraction grating has 8900 slits across 1.20 cm. If light with
a wavelength of 500 nm is sent through it, how many fraction of the original light energy was picked up by the detector? Assume that for each direction of travel all the energy is in the central diffraction peak.

93 In June 1985, a laser beam was sent out from the Air Force

105 Show that

Optical Station on Maui, Hawaii, and reflected back from the shuttle
 Discovery as it sped by 354 km overhead. The diameter of the central

m 93 In June 1985, a laser beam was sent out from the Air Force

Optical Station on Maui, Hawaii, and reflected back from the shuttle
 Discovery as it sped by 354 km overhead. The diameter of the central

maximum of the b 93 In June 1985, a laser beam was sent out from the Air Force

Optical Station on Maui, Hawaii, and reflected back from the shuttle

Discovery as it sped by 354 km overhead. The diameter of the central

ima (ex

maximum o 93 In June 1985, a laser beam was sent out from the Air Force **105** Show that a grating made up of
Optical Station on Maui, Hawaii, and reflected back from the shuttle opaque strips of equal width eliminate
Discovery as i 93 In June 1985, a laser beam was sent out from the Air Force 105 Show
Optical Station on Maui, Hawaii, and reflected back from the shuttle opaque strip
Discovery as it sped by 354 km overhead. The diameter of the central and the beam wavelength was 500 nm. What is the effective diameter 2.00μ m and onto a screen that is 2.00 m away. On the screen, what of the laser aperture at the Maui ground station? (*Hint*: A laser beam is the dista 93 In June 1985, a laser beam was sent out from the Air Force 105 Show
Optical Station on Maui, Hawaii, and reflected back from the shuttle opaque strip
Discovery as it sped by 354 km overhead. The diameter of the central 93 In June 1985, a laser beam was sent out from the Air Force

Optical Station on Maui, Hawaii, and reflected back from the shuttle

consequence the central

maximum of the beam at the shuttle position was said to be 9.1 **93** In June 1985, a laser beam was sent out from the Air Force **105** Show that Optical Station on Maui, Hawaii, and reflected back from the shuttle opaque strips of *Discovery* as it sped by 354 km overhead. The diameter 93 In June 1985, a laser beam was sent out from the Air Force **105** Show that a gratino Optical Station on Maui, Hawaii, and reflected back from the shuttle paque strips of equal with Discovery as it sped by 354 km overhe Optical Station on Maui, Hawaii, and reflected back from the shuttle opaque strips of *Discovery* as it sped by 354 km overhead. The diameter of the central ima (except $m =$ maximum of the beam at the shuttle position was Discovery as it sped by 354 km overhead. The diameter
maximum of the beam at the shuttle position was said
and the beam wavelength was 500 nm. What is the effec
of the laser aperture at the Maui ground station? (*Hint:*
s

Monochromatic light that is incident normally is diffracted
Monochromatic light that is incident normally is diffracted
within the first side needs of the differential application of different set different

the central maximum of the diffraction pattern increases by a and the beam wavelength was 500 nm. What is the effective diameter
of the laser aperture at the Maui ground station? (*Hint:* A laser beam
spreads only because of diffraction; assume a circular exit aperture.)
94 A diff **94** A diffraction grating 1.00 cm wide has 10 000 parallel slits.

Monochromatic light that is incident normally is diffracted

through 30° in the first order. What is the wavelength of the light?
 95 SSM If you doub **95 SSM** If you double the width of a single slit, the intensity of

96 When monochromatic light is incident on a slit 22.0 μ m wide, the first diffraction minimum lies at 1.80° from the direction of the incident light.What is the wavelength?

94 A diffraction grating 1.00 cm wide has 10 000 parallel slits.

Monochromatic light that is incident normally is diffracted
 95 SSM If you double the width of a single slit, the intensity of

the central maximum o Monochromatic light that is incident normally is diffracted
through 30° in the first order. What is the wavelength of the light?
 95 SSM If you double the width of a single slit, the intensity of

the central maximum craft's air intake port.What is the effective diameter of the lens as determined by diffraction consideration alone? Assume $\lambda = 550$ nm. the central maximum of the diffraction pattern increases by a

factor of 4, even though the energy passing through the slit only

is normally incident on a gra

doubles. Explain this quantitatively!
 98 Suppose that is factor of 4, even though the energy passing through the sit only

so an equality incident on a solution windinfumulities at 1.80° from the direction of the grating spaci

incident light. What is the wavelength?

incident **96** When monochromatic light is incident on a slit 22.0 μ m wide,
the first diffraction minimum lies at 1.80° from the direction of the
incident light. What is the wavelength?
97 A spy satellite orbiting at 160 km ab the first diffraction minimum lies at 1.80° from the direction of the
incident light. What is the wavelength?
 97 A spy satellite orbiting at 160 km above Earth's surface has a lens

some a considered into 3.6 m and can

from the viewer puts them at the Rayleigh limit of resolution?

incident light. What is the wavelength?
 97 A spy satellite orbiting at 160 km above Earth's surface has a lens

with a focal length of 3.6 m and can resolve objects on the ground as

small as 30 cm. For example, it can **97** A spy satellite orbiting at 160 km above Earth's surface has a lens Eq. 36-19 red
with a focal length of 3.6 m and can resolve objects on the ground as
for such a slit.
small as 30 cm. For example, it can easily meas der that is overlapped by another order? (b) What is the highest $\frac{113}{113}$ An acoustic double-slit system (of slit separation d and order for which the complete spectrum is present? onsideration alone? Assume $\lambda = 550$ nm.

pattern.

points are separated by 2.0 cm. If they are

a pupil opening of 5.0 mm, what distance

both wavelengths the Rayleigh limit of resolution?

gentle of reflecting

gentles

100 A diffraction grating has 200 rulings/mm, and it produces an intensity maximum at $\theta = 30.0^{\circ}$. (a) What are the possible wavelengths of the incident visible light? (b) To what colors do they correspond?

101 SSM Show that the dispersion of a grating is $D = (\tan \theta)/\lambda$. Take both interfered

102 Monochromatic light (wavelength $= 450$ nm) is incident perpendicularly on a single slit (width $= 0.40$ mm). A screen is placed **99** A diffraction grating has 200 lines/mm. Light consisting of a

continuous range of wavelengths between 550 nm and 700 nm is

incident perpendicularly on the grating. (a) What is the lowest or-

der that is overlapped continuous range of wavelengths between 550 nm and 700 nm is
time (400–700 nm) can be produced
eder that is overlapped by another order? (b) What is the highest
order for which the complete spectrum is present?
113 An a is the distance from the slit to the screen? (Hint: The angle to either minimum is small enough that sin $\theta \approx \tan \theta$. (b) What is the distance on the screen between the first minimum and the third minimum on the same side of the central maximum? tensity maximum at $\theta = 30.0^{\circ}$. (a) What are the possible wavelengths occur in the double-slit diffraction
of the incident visible light? (b) To what colors do they correspond?
 101 SSM Show that the dispersion of of the incident visible light? (b) To what colors do they correspond?
 101 SSM Show that the dispersion of a grating is $D = (\tan \theta)/\lambda$. Take both interference and d
 102 Monochromatic light (wavelength = 450 nm) is incid 102 Monochromatic light (wavelength = 450 nm) is incident per-
pendicularly on a single slit (width = 0.40 mm). A screen is placed
parallel to the slit plane, and on it the distance between the two
minima on either side

(1) that the first and second maxima for each wavelength appear at **The third order for the 600 nm** light be a missing order. (a) What signal parallel to the slit plane, and on it the distance between the two
minima on either side of the central maximum is 1.8 mm. (a) What
is the distanc should be the slit separation? (b) What is the smallest individual **114** Two emission lines have wavelengths λ and $\lambda + \Delta \lambda$, respectionally individual slit width that can be used? (c) For the values calculated in (a) and is the distance from the slit to the screen? (*Hint*: The angle to ei-
ther minimum is small enough that sin $\theta \approx \tan \theta$.) (b) What is the
distance on the screen between the first minimum and the third
minimum on the same maxima produced by the grating? 103 Light containing a mixture of two wavelengths, 500 and
600 nm, is incident normally on a diffraction grating. It is desired
(1) that the first and second maxima for each wavelength appear at
 $\theta \le 30^{\circ}$, (2) that th **beams** are produced for 0.100 mm and the plane separation is 0.250 nm. It is observed that seems are produced for the incident and scattered beams?
 beams are produced for the 600 nm light be a missing order. (a) What $\theta \le 30^{\circ}$, (2) that the dispersion be as high as possible, and (3) that

104 A beam of x rays with wavelengths ranging from 0.120 nm to 0.0700 nm scatters from a family of reflecting planes in a crystal. between the incident and scattered beams?

105 Show that a grating made up of alternately transparent and opaque strips of equal width eliminates all the even orders of maxima (except $m = 0$). 0).

106 Light of wavelength 500 nm diffracts through a slit of width **2.00** PROBLEMS **1115**

2.00 Show that a grating made up of alternately transparent and

opaque strips of equal width eliminates all the even orders of max-
 106 Light of wavelength 500 nm diffracts through a slit of wi is the distance between the center of the diffraction pattern and the third diffraction minimum?

105 Show that a grating made up of alternately transparent and opaque strips of equal width eliminates all the even orders of maxima (except $m = 0$).
 106 Light of wavelength 500 nm diffracts through a slit of width within the first side peak of the diffraction envelope and diffrac-**105** Show that a grating made up of alternately transparent and opaque strips of equal width eliminates all the even orders of maxima (except $m = 0$).
 106 Light of wavelength 500 nm diffracts through a slit of width 2 is the ratio of slit separation to slit width?

intensity of is the ratio of slit separation to slit eases by a

the slit only

is normally incident on a grating

value of the grating spacing d, t

of the grating spacing d, t

overlap.

Of the grating spacing d, t

ove 108 White light (consisting of wavelengths from 400 nm to 700 nm) opaque strips of equal width eliminates all the even orders of max-
ima (except $m = 0$).
 106 Light of wavelength 500 nm diffracts through a slit of width

2.00 μ m and onto a screen that is 2.00 m away. On the screen ima (except $m = 0$).
 106 Light of wavelength 500 nm diffracts through a slit of width 2.00 μ m and onto a screen that is 2.00 m away. On the screen, what is the distance between the center of the diffraction pattern overlap. een that is 2.00 m away. On the screen, what

in the center of the diffraction pattern and

nimum?

terference pattern, there are 8 bright fringes

eak of the diffraction envelope and diffrac-

vith two-slit interference is the distance between the center of the diffraction pattern and
the third diffraction minimum?
 107 If, in a two-slit interference pattern, there are 8 bright fringes

within the first side peak of the diffraction env the third diffraction minimum?
 107 If, in a two-slit interference pattern, there are 8 bright fringes

within the first side peak of the diffraction envelope and diffrac-

tion minima coincide with two-slit interferenc within the first side peak of the diffraction envelope and diffraction minima coincide with two-slit interference maxima, then what is the ratio of slit separation to slit width?
 108 White light (consisting of waveleng

109 If we make $d = a$ in Fig. 36-50, the two slits for such a slit.

half-width of the lines in a grating's diffraction pattern.

111 Prove that it is not possible to determine both wavelength of incident radiation and spacing of reflecting planes in a crystal by measuring the Bragg angles for several orders.

Figure 36-50 Problem 109.

112 How many orders of the entire visible spectrum (400–700 nm) can be produced by a grating of 500 lines/mm?

order for which the complete spectrum is present?
 Solution is the complete spectrum is present?
 Solution is dividend in the complete spectrum is present?
 Solution is dividend in the complete specific specific speci (tan θ)/ λ . Take both interference and diffraction effects into account. Bragg angles for sever

screen 550 nm and 700 nm is

ig. (a) What is the lowest or-

der? (b) What is the highest

is present?

also wavelengths

and is present?

also wavelengths

are the possible wavelengths

at colors 113 An acoustic double-slit system (of slit separation d and **110** Derive Eq. 36-28, the expression for the half-width of the lines in a grating's diffraction pattern.
 111 Prove that it is not possible to determine both wavelength of incident radiation and spacing $\frac{1}{a}$ Bran 110 Derive Eq. 36-28, the expression for the half-width of the lines in a grating's diffraction $\frac{1}{4}$

111 Prove that it is not possible to determine both wavelength of incident radiation and spacing

of reflecting pl half-width of the lines in a grating's diffraction

pattern.

111 Prove that it is not possible to determine

both wavelength of incident radiation and spacing

Bragg angles for several orders.

112 How many orders of the occur in the double-slit diffraction pattern at large distances as the phase difference between the speakers is varied from zero to 2π .

Figure 36-51 Problem 113.

grating spectrometer is given approximately by **Figure 36-51** Problem 113.
 114 Two emission lines have wavelengths λ and $\lambda + \Delta \lambda$, respectively, where $\Delta \lambda \ll \lambda$. Show that their angular separation $\Delta \theta$ in a grating spectrometer is given approximately by $\Delta \$ tively, where $\Delta \lambda \ll \lambda$. Show that their angular separation $\Delta \theta$ in a grating spectrometer is given approximately by
 $\Delta \theta = \frac{\Delta \lambda}{\sqrt{\lambda}}$.

$$
\Delta \theta = \frac{\Delta \lambda}{\sqrt{(d/m)^2 - \lambda^2}},
$$

where d is the slit separation and m is the order at which the lines higher orders than the lower orders.